Estimation of the Isotropic Neutrino Flux from Quasi Stellar Radio Sources

Julia K. Becker

Öu - Nat - Ex - FYS01D

Supervisors: Wolfgang Rhode, Peter L. Biermann
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# Contents

1 Astroparticle Physics and Cosmology ................................................. 4
  1.1 A glimpse into astroparticle physics ............................................ 4
    1.1.1 The Cosmic Ray spectrum .................................................. 5
    1.1.2 Acceleration processes ..................................................... 6
    1.1.3 Neutrino production ....................................................... 8
    1.1.4 Neutrino spectra predictions ............................................ 10
  1.2 Active Galactic Nuclei (AGN) .................................................. 12
    1.2.1 Basic features ............................................................ 12
    1.2.2 Classification criteria .................................................. 12
    1.2.3 Radiation processes ...................................................... 13
    1.2.4 AGN classes ............................................................... 14
    1.2.5 Morphology ............................................................... 16
    1.2.6 Unification ............................................................... 17
    1.2.7 Neutrino production in AGN ............................................ 18
    1.2.8 Jet-disk symbiosis ....................................................... 18
  1.3 Cosmology ................................................................................. 23
    1.3.1 Cosmological parameters .................................................. 23
    1.3.2 Cosmological distances ................................................... 24
    1.3.3 The Comoving Volume ..................................................... 25
    1.3.4 Cosmic microwave background ........................................ 26
    1.3.5 Measuring cosmological parameters ................................... 27

2 Experiments ................................................................................. 31
  2.1 ... in water and ice ................................................................. 31
  2.2 Airshower arrays ................................................................. 33
  2.3 ... in salt ............................................................................... 34
3 A model for the neutrino energy spectrum from quasars 36
3.1 Ingredients ................................................ 36
3.1.1 The generic neutrino flux .............................. 38
3.1.2 The AGN density ....................................... 40
3.1.3 Cosmological tools ..................................... 47
3.1.4 Integration limits ...................................... 48
3.2 Calculation ................................................ 49
3.3 Constraints and limits on the flux ......................... 51
3.4 Results and Conclusions ................................ 52
3.5 Outlook .................................................. 58

A Overview of the calculation results 60
List of Figures

1.1 The all particle Cosmic Ray spectrum .......................... 5
1.2 The Hillas plot .................................................. 7
1.3 Second order Fermi acceleration ............................... 8
1.4 First order Fermi acceleration ................................. 8
1.5 Neutrino spectra Predictions. ................................. 10
1.6 Synchrotron radiation .......................................... 13
1.7 Scheme of the continuum of a typical AGN spectrum [CO96] .... 14
1.8 AGN scheme ..................................................... 18
1.9 NGC-4261 ......................................................... 19
1.10 Particle production in AGN. .................................. 20
1.11 Distance measures .............................................. 26
1.12 COBE CMB measurements .................................... 27
1.13 CMB spectrum .................................................. 28
1.14 Stretch corrected Hubble plot for SN-Ia ........................ 29
1.15 The cosmological diagram .................................... 30
2.1 The AMANDA detector .......................................... 32
2.2 Scheme of a horizontal tau neutrino airshower [Cro01] .......... 33
3.1 Windhorst plot .................................................. 39
3.2 Luminosity function at 0.5 GHz [Sch72], fitted by a power law. .... 41
3.3 RLF at 1.4 GHz. ............................................... 42
3.4 Monochromatic total quasar luminosity. ....................... 44
3.5 Distance distributions. ........................................ 45
3.6 Spatial AGN distribution [MHS00]. ............................ 47
3.7 Madau Star Formation Rate. .................................. 48
3.8 Atmospheric neutrino flux. ................................... 51
3.9  z-dependence of the integrand. .............................. 52
3.10 Variation of the upper redshift limit. ........................ 53
3.11 Flux in the Miyaji model. ...................................... 55
3.12 Flux in Madau’s model. ......................................... 56
3.13 Lower flux limit. ................................................ 57
List of Tables

1.1 The AGN zoo ............................................. 16
1.2 Relation between radio and disk luminosity .............. 21
1.3 Current cosmological parameters .......................... 28
3.1 Comparison of the two models used ......................... 46
Preface

The history of neutrino-physics began in the 1930th, when Wolfgang Pauli tried to explain an energy discrepancy in the beta-decay by postulating a new particle - the neutrino. Today, this theory is well established and the neutrino is part of the standard particle model. The history of astrophysics began much earlier, when people tried to motivate the movement of celestial bodies physically and abandoned the idea of a solely religious motivation. Kopernikus and Galilei as well as Kepler and Brahe initiated the process of understanding celestial mechanics. Although the standard models of particle physics and cosmology do not comprise the whole truth, they describe our reality quite well. It is the task of today’s and tomorrow’s physicists to solve the rest of the puzzle.

Physicists of the past and of the presence enabled us to combine neutrino-physics with astrophysics and thus to practice neutrino-astrophysics. The following thesis aims at two points: To understand astronomical objects by regarding their neutrino energy spectrum as well as their neutrino production rate and the other way around observing neutrinos from extragalactic sources to examine the behavior of ultra high energy neutrinos. More detailed, the aim of this thesis is the prediction of the neutrino flux from radio quasars. Here, the idea is quite simple: Taking the generic energy spectrum of a single source, multiplying it with the number of quasars in the Universe and considering the decrease of the flux by cosmological distance, a flux at a certain redshift and luminosity is given. Integrating the result over redshift and luminosity gives the neutrino flux at a certain energy. The practical realization of the calculation is quite more complicated: The number of Active Galactic Nuclei (AGN) in the Universe is strongly dependent on the luminosity and on the redshift. The errors of the used models are quite high, especially for higher redshifts. Furthermore, the models are partly given with a set of cosmological parameters which is today known as incorrect. An Einstein-de-Sitter Universe was assumed in most of the cases. In the recent years, these parameters have been determined with an astonishing accuracy and the Einstein-de-Sitter model could be excluded. It was therefore important to find the models with the most realistic set of cosmological parameters. To get an estimate of the lower bound of the neutrino flux from quasars, the errors of the distribution functions are concerned in the calculations.

Current neutrino-experiments, in particular AMANDA, measure the atmospheric neutrino flux up to an energy of approximately 100 TeV [Gee03]. Extraterrestrial sources have not yet been observed, since the atmospheric neutrino flux dominates over the extraterrestrial flux at the measured energies. The calculated model of the quasar neutrino flux predicts a significant contribution to the all neutrino spectrum at slightly higher energies. Even the lower limit of the quasar flux could be measured by ICECUBE, a next generation telescope which is a successor experiment of the AMANDA experiment at the geographical South Pole.
Förord


Aktuella neutrinoexperiment, speciellt AMANDA, mäter det atmosfäriska neutrinoflödet upp till en energi av 100 TeV [Gee03]. Kosmologiska källor har inte observerats än eftersom det atmosfäriska flödet dominerar vid dessa energier. Modellen som beräknas här förutsätter att kvasarer bidrar signifikant vid högre energier. Även den undre gränsen för neutrinoflödet är inom räckhåll för ICECUBE, som är ett planerade neutrinooptiskt teleskop vid Sydpolen som ska följa upp det nuvarande AMANDA experimentet.
Vorwort


Aktuelle Neutrinobeobachtungen, insbesondere AMANDA, messen den atmosphärischen Neutrinofluss bis zur Energie von ca. 100 TeV [Gee03]. Extraterrestrische Quellen werden bisher nicht beobachtet, weil in dem gemessenen Energiebereich der atmosphärische Fluss dominiert. In dem hier berechneten Modell wird vorhergesagt, dass allein von Quasaren bei höheren Energien ein signifikanter Beitrag zu erwarten ist. Die untere Grenze des Flusses liegt im Bereich der Sensitivität von ICECUBE, einem Neutrino-Teleskop der nächsten Generation, was als Nachfolgeprojekt von AMANDA am Südpol errichtet werden soll.
Chapter 1

Astroparticle Physics and Cosmology

1.1 A glimpse into astroparticle physics

In 1785, Coulomb found out that air is electrically conducting. The discovery of radioactivity by Bequerell (1896) and Curie (1898) [Kra96] helped to explain the phenomenon since ionizing radiation was found coming from rocks containing uranium. Later in 1912, Viktor Hess conducted experiments in hot air balloons and discovered that the ionizing radiation was increasing with the height of the balloon in the atmosphere. Further experiments made by Kohlhörster (1914) and Millikan (1926, [Mil26]) reaching a height of 15 km confirmed the extraterrestrial origin of the radiation. It was Millikan who introduced the term Cosmic Rays (CRs) for the radiation which is still in use. Since then, the Cosmic Ray spectrum has been examined in detail and it has been subdivided into two particle types: The primary Cosmic Rays are particles which are produced in distant sources and reach the Earth directly from outer space. Secondary Cosmic Rays are induced by interactions between primary (or even secondary) CRs and molecules in the atmosphere. In the early years, a lot of progress in particle physics was made through the examination of the CRs. The discovery of the muon (1937, [SS37]) and of several mesons (e.g. the pions and kaons) and hadrons (e.g. the Delta resonance and Ξ) was made in CR experiments before the field of particle physics was dominated by accelerator physics. The use of accelerators then became so successful that cosmic ray particle physics stagnated for a long while. Today it is known that the Cosmic Ray energy spectrum exceeds the upper energy limit for accelerator physics by several orders of magnitude. While man-made accelerators can reach energies of several TeV, cosmic accelerators produce particles with energies up to $10^9$ TeV. Today, elementary particle physics and astroparticle physics complement one another since the accurate results gained in accelerator physics at low energies are used in CR physics while knowledge about ultra high-energy (UHE) particles is achieved by observing cosmic accelerators. Also, question of non-vanishing neutrino masses has been examined closely with much progress by CR physics: Neutrino oscillations have been observed by SNO recently [SNO02]. If oscillating from one flavor to another, at least two
neutrino flavors must have a mass $m_{\nu_i} > 0$.

Besides the neutrino oscillation/mass-question, the fundamental questions being asked in modern astroparticle physics are

- What is the origin of the CRs?

- What does the energy spectrum of the CRs look like?

- What are the acceleration processes?

The current status in answering these questions will be presented here very shortly.

### 1.1.1 The Cosmic Ray spectrum

Today it is known that the Cosmic Ray spectrum at energies $E > 1$ GeV consists of approximately 90% protons, 10% $\alpha$–particles, 1% heavy nuclei and 1% leptons [Rac92]. The energy spectrum is shown in figure 1.1. The particle flux is given in units $[\Phi] = 1/(\text{GeV} \cdot \text{s} \cdot \text{sr} \cdot \text{cm}^2)$. In figure 1.1, the flux is multiplied by $E^{2.75}$ so that the slopes of the different parts of the spectrum are sufficiently flat to have a good presentation. It can be clearly seen that the spectrum has two kinks. The first one at approximately $E \approx 10^{15}$ eV is called the knee while the second one at $E \approx 10^{19}$ eV is referred to as the ankle. All three parts can mathematically be described by a power law, $\Phi \propto E^{-7}$. The spectral indices for the different parts of the spectrum
are \cite{Wie98}

\[\gamma \approx \begin{cases} 
2.67 & \text{for } E < 10^{15} \text{ eV} \\
3.10 & \text{for } 10^{15} \text{ eV} < E < 10^{19} \text{ eV} \\
2.75 & \text{for } 10^{19} \text{ eV} < E < 10^{21} \text{ eV}.
\end{cases} \tag{1.1}\]

Different particles are produced with slightly different indices, so that the numbers above can be regarded as a mean value for all particles. An ansatz to explain the power law spectra is \emph{diffuse shock acceleration} which will be explained in section 1.1.2. The spectral index is determined through the kinematics of the acceleration process so that different objects produce fluxes with different spectral indices. That explains the kinks in the spectrum since calculations for supernovae into the interstellar medium (ISM-SN) as well as pulsar winds predict acceleration up to \(10^{15}\) eV while explosions of heavy star supernova into their own previous wind\(^1\) can accelerate particles up to \(10^{19}\) eV. A mystery in astroparticle physics are the events above \(\approx 5 \times 10^{19}\) eV. A theory developed by Greisen, Zatsepin and Kuzmin (GZK, \cite{Gre66}) predicts that particles with energies \(E > 5 \times 10^{19}\) eV will interact with the Cosmic Microwave Background (CMB) and therefore have an approximative mean free path of \(< 50\) Mpc. This implies that the any source producing energies at \(E > 5 \times 10^{19}\) eV should not be too far away. However, about 20 events with energies above this limit have been detected by AGASA. It has not yet been completely understood why there are such high-energy events. One idea is that Active Galactic Nuclei (AGN) of the type FR-II, which are the objects with the strongest and most effective shockwaves in the Universe, may be able to produce particles which exceed the GZK limit\(^2\). Energies up to \(10^{20}\) eV can be reached so that the CR flux above the ankle could be explained by particle acceleration in AGN. Other possible sources for ultra high-energy events are Gamma Ray Bursts (GRB) and Topological Defects (TD).

### 1.1.2 Acceleration processes

The first step towards interpreting the Cosmic Ray spectrum is examining the possible acceleration processes. Cosmic acceleration of particles is explained by the \emph{Bottom-Up} scenarios which can be divided into two groups: There are models describing objects accelerating particles through a one-step acceleration process. The other possibility is shock-wave acceleration where many small acceleration processes are considered with an final effective acceleration found by using statistical methods. Regardless of the acceleration process, the maximum energy that can be reached by a particle of charge \(Z e\) is given by \cite{Hil84}

\[E_{\text{max}} = \beta \cdot Z \left( \frac{B}{1\mu G} \right) \left( \frac{R}{1 \text{ kpc}} \right). \tag{1.2}\]

Here, \(B\) is the magnetic field strength of the shock front, \(R\) is the size of the particle accelerating object and \(\beta\) is the shock velocity in units of the speed of light. The equation is visualized in figure 1.2, the so called Hillas plot. The lines in the plot represent the energy limits for various particles. Objects below the lines are not able

---

\(^1\)These SNs will from now on be called supernova remnants.

\(^2\)For a detailed description of AGN see section 1.2.
Figure 1.2: The Hillas plot showing various objects. The lines represent proton acceleration (upper: $\beta = 1/300$ and middle: $\beta = 1$) and iron acceleration with $\beta = 1$. Objects below the lines cannot accelerate the corresponding nucleon to energies above the GZK cutoff ($E > 5 \cdot 10^{19}$ eV) [Arg00].

to accelerate the corresponding particle to energies above the GZK cutoff. AGN are a candidate for the so called super-Greisen events.

The first approach to stochastic acceleration was made by E. Fermi. In First order Fermi acceleration particles are accelerated in a plane shock front moving with a velocity $V_s$ while the shocked gas moves with $V_p$ (see figure 1.4). It describes an acceleration process in which the particle gains energy in every acceleration cycle. Second order Fermi acceleration considers a moving cloud of plasma in which the particle can also lose energy in acceleration cycles [Gai90]. The two different acceleration processes are presented in figure 1.3 and 1.4. First order Fermi acceleration is also known as diffuse shock acceleration. Fermi acceleration leads to the observed power law spectrum. To accelerate particles to the energies found in the Cosmic Ray spectrum, strong shock waves are necessary like the ones produced in supernova explosions. The spectral index is determined by the shock kinematic. The maximum

---

3Second order Fermi acceleration was Fermi's original approach to stochastic shock acceleration. First order Fermi acceleration was introduced in the 1970th.
Figure 1.3: The original Fermi theory (Second order Fermi acceleration). Interaction of a cosmic particle of energy \( E_1 \) in a cloud moving with speed \( V \) [Pro98].

Figure 1.4: First order Fermi acceleration. The shock front is considered to be plain, moving with a velocity of \( V_s \). The ejected matter has a velocity of \( V_p \) [Pro98].

reachable energy is determined through the strength of the magnetic field. Acceleration in supernova shocks can only result in energies up to \( 10^{15} \) eV which explains the change of the spectral index at this energy. Particle acceleration in pulsar wind shocks in binary systems containing neutron stars is also possible up to \( 10^{15} \) eV. The main part of the Cosmic Ray spectrum beyond the knee is assumed to be produced by supernova remnants. These objects can accelerate particles up to approximately \( 10^{19} \) eV. However, events above \( E = 10^{19} \) eV cannot result from particle acceleration within the Galaxy so that these events are assumed to be extragalactic. Beyond the ankle, AGN, GRB or TD are believed to be the particle sources as it will be explained in section 1.2.

To distinguish between first order and second order Fermi acceleration, the different geometries have to be considered. Explicit calculations can be found in [Gai90] and [Pro98]. Two conclusions can be drawn from Fermi acceleration

- The time it takes to accelerate for a particle up to a certain energy increases with the energy of the initial particle.
- Limiting a certain Fermi accelerator to a lifetime \( T_{acc} \), it is characterized by a maximum energy per particle that it can produce.

1.1.3 Neutrino production

Neutrinos can be produced in weak decays subsequent to the product of hadrons in hadronic interactions of protons with a target. The target can be matter or photon fields [GHS95]. In particular, interactions of protons with photons are relevant for this thesis since protons from the footring of the jet of an AGN are assumed to interact with photons from the disk (see section 1.2):

\[
\begin{align*}
p \gamma & \rightarrow \Delta^+ \rightarrow n \pi^+ \\
n \gamma & \rightarrow \Delta^0 \rightarrow p \pi^-. 
\end{align*}
\]

The pions and the neutron then produce neutrinos through the decays

\[
\begin{align*}
\pi^+ & \rightarrow \mu^+ \nu_\mu \rightarrow e^+ \nu_e \overline{\nu}_\mu \nu_\mu 
\end{align*}
\]
\[ \begin{align*}
\pi^- & \longrightarrow \mu^- \bar{\nu}_\mu \\
n & \longrightarrow p e^- \bar{\nu}_e.
\end{align*} \]

The neutrino flux measured on Earth can be subdivided into fluxes of different origins [Rho02].

The atmospheric neutrino flux is produced by primary charged Cosmic Rays which interact with nuclei from the upper atmosphere. While the spectral index of the primary hadrons is\(^4\) \(\gamma \approx 2.67\) [Rho02], the atmospheric neutrino spectrum is one power steeper since neutrinos are mainly produced by meson decay and the lifetimes of the produced mesons are long enough to interact with molecules in the atmosphere. Besides the light quark mesons (consisting only of u, d and s-quarks) there is a small fraction of mesons being produced that includes heavy (c) quarks. They decay immediately without interaction. Therefore, their spectral index is expected to follow the primary spectrum. These mesons are also called prompt [Gai90]. Furthermore, the flatter the zenith angle for the incoming primary is, the longer the path of the meson within the atmosphere before hitting the Earth. This implies that at sufficiently high energies the intensity of the meson decay product increases with the zenith angle.

Extraterrestrial neutrinos could be observed at high energies where the atmospheric spectrum is suppressed. The spectrum of the extraterrestrial neutrinos may have a galactic or extragalactic origin:

- **Solar neutrinos** are observed in an energy range of \(\sim\) MeV. By observing solar neutrinos, the standard solar model can be tested and neutrino oscillations could be observed recently [SNO02].

- **Galactic neutrino sources** are supernovae and SN remnants as well as strong X- and \(\gamma\)-ray sources (e.g. X-ray binaries and neutron stars). Neutrinos can also be produced indirectly through interactions of the charged CRs with the galactic plane.

- **Extragalactic neutrino sources** are able to accelerate protons to ultra high energies which produce ultra high energy neutrinos in \(p - \gamma\) and \(p - p\) interactions as described previously. For instance, ultra high energies are assumed to be reached by particles in Active Galactic Nuclei (AGN) which are galaxies with a supermassive black hole in the center and an accretion disk. Perpendicular to the disk, there are two relativistic jets presumably producing ultra high energy particles up to \(10^{21}\) eV [KRB98]. Another possible source are Gamma Ray Bursts (GRBs): These emit a lot of energy in the form of \(\gamma\) radiation on short scales\(^5\) presumably from compact sources. GRBs are not yet completely understood but there are different models trying to explain the phenomena. One idea is that the GRBs are produced by a fireball which is the product of two colliding neutron stars [Pug00]. In this model, low energy neutrinos are produced from the fireball and time delayed neutrinos result from beam dumps

\(^4\)Assuming energies \(E < 10^{15}\) eV.
\(^5\)A burst can endure between seconds and hours.
of accelerated protons. Another possibility to explain the origin of GRBs is to assume that they are produced during the collapse of a massive star [Dar03]. Topological Defects (TDs), relics from the time shortly after the Big Bang, can also be sources for neutrinos [Rho02]. These could be magnetic monopoles and domain walls that were generated in the time after the Big Bang when three forces were still unified6. If there are TDs still remaining, these would decay hadronically so that they will finally produce photons and neutrinos.

1.1.4 Neutrino spectra predictions

![Diagram of neutrino spectra](image)

Figure 1.5: Various predictions of cosmic neutrino spectra taken from [LM00]. The flux is multiplied with $E^2$.

There are various models for the flux from different sources compiled in [LM00]. Some of the predictions and bounds will be presented here as well as the current bound on the atmospheric neutrino flux. Figure 1.5 shows a number of neutrino flux models and the prediction of the contribution of various source types.

The prediction of the atmospheric neutrinos includes a charm production model [TIG96]. The darkest shaded region presents the flux from the galactic disk towards the center (upper boundary) and the poles (lower boundary). The upper bound of the light shaded region comes from unresolved extragalactic sources from which $\gamma$ rays and nucleons escape freely (curved) and from which only $\gamma$ rays escape (straight). The lower boundary is due to CR storage in galaxy clusters.

The numbered lines represent the following neutrino flux models:

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6The theories concerning the unification of the three forces (excluding the gravitation) are called *Grand Unified Theories - GUT*. 
1. A model by Nellen et al. [Nel93] gives a prediction for $p - p$ interactions in the core of AGN.

2. Stecker and Salamon [SS96] developed a neutrino flux model $p - \gamma$ interactions in AGN cores.

3. Mannheim, Protheroe and Rachen [RPM99] present a model of neutrino production through $p - \gamma$ interactions in extragalactic photoproduction sources.

4. A further model by Mannheim [Man95] is given for $p - \gamma$ interactions in blazar jets. UHE events are produced here though neutron escape.

5. Rachen, Biermann [RB93] examined $p - \gamma$ interactions due to UHE Cosmic Rays from radio galaxies traveling through the CMB.

6. Proton proton interactions in host galaxies of blazars are examined by Mannheim in [Man95].

7. Gamma Ray Bursts are a candidate neutrino source and the GRB flux is predicted by Waxman and Bahcall [WB97].

8. A topological defect model is presented by Sigl et al. [Sig98] as well as by Birkel and Sakar [BS98].

Recent experimental results on the atmospheric neutrino flux are given by AMANDA data analyzed in [Gee08] and presented in figure 3.8. As expected, the spectrum follows a power law of $\sim E^{-3.7}$. 
1.2 Active Galactic Nuclei (AGN)

Looking at distant objects always implies looking back in time. AGN are found at distances as far as $z \approx 6$. Therefore, AGN carry information about an early stage of the Universe. Looking back in time also means higher energy densities and for this reason, AGN release ultra high energy particles. Gamma radiation from AGN has been detected by experiments such as Whipple, HEGRA and CANGAROO [CW99, HEG02, MC01] which observe photons at energies $E > 1$ TeV. Future experiments, aiming a lower energy limit $E > 20 – 50$ GeV are HESS and MAGIC [Ste00, PM99]. A matter of particular interest is the observation of photon emission by EGRET [L+$99$] which shows that about half of the ultra high energy sources are AGN. This is astonishing because objects in the galactic disk should appear much brighter since they are closer to Earth. Another possibility (in the near future) is the detection of AGN neutrinos with detectors such as AMANDA, ICECUBE, Antares, Auger, Baikal [CA02, Mon03, B+$02$, DB02] and more. Until today, many of these experiments are still in an early stage of operation (if already running) and yet too small for the detection of rare events such as the detection of ultra high energy neutrinos. Therefore, the estimation of the neutrino flux from AGN is of particular interest for experimental analysis. In this section, AGN will be classified by their features and a unified model for all AGN classes will be given.

1.2.1 Basic features

Looking at the history of Active Galactic Nuclei, the first objects discovered and later classified as AGN were Seyfert galaxies. These are spiral galaxies with a very bright nucleus first detected by Carl Seyfert in 1943. Little by little, other types of galaxies with bright cores and a strong radio appearance were discovered. Today, these apparently different galaxies are believed to represent the same type of galaxy only seen from different angles. The unified model for all these galaxies with a bright core is basically the following: Active Galactic Nuclei (AGN) are described today as objects with a central engine, which is presumed to be a black hole. A rotating black hole with matter around forms an accretion disk. Furthermore, there are two jets perpendicular to the disk. So far, the appearance of the two jets is not completely understood. AGN are a possible source of UHE particles because of the relativistic jet which can produce enormous particle energies assuming first order diffuse Fermi acceleration (see section 1.1.2).

1.2.2 Classification criteria

Several aspects are considered when classifying different types of AGN: There is a significant non-thermal component in each AGN spectrum which follows a power law ($\sim \nu^{-\alpha}$). A power law often indicates synchrotron radiation as the dominant radiation process which will be explained in section 1.2.3. The photons are boosted

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$^7$Beazor jets, for instance, are pointing directly towards Earth while both jets are seen from the side for quasars.
Figure 1.6: Synchrotron radiation leads to a power law spectrum as shown in this figure since the single electron spectra overlap so that the integral distribution is a power law [CO96].

to higher energies by inverse Compton scattering. AGN classes are differentiated by their spectral index, that is $\alpha > 0.5$ means “steep” and $\alpha < 0.5$ means “flat”. Spectra with a negative index are called “inverse”.

Another distinction is the power emitted at radio frequencies. Of course, all AGN have a very high radio luminosity compared to normal galaxies. Nevertheless, there are AGN which are less powerful (“radio quiet”) than others (“radio-loud”).

Since there are gas clouds around AGN which absorb photons coming from the AGN, the emission lines (of the gas) are broadened due to the Doppler effect: Depending on the velocity of the particles in the absorbing gas, the emission lines are narrow (cold gas) or broad (hot gas). AGN are differentiated with respect to whether they have broad emission lines (higher velocity) or narrow emission lines (lower velocities). Finally, AGN occur as spiral galaxies and also as elliptical galaxies.

1.2.3 Radiation processes

Beneath the (thermal) blackbody spectrum of a regular star or galaxy, there is a significant non-thermal component found in AGN spectra. The monochromatic energy flux $F_\nu$ detected from the non-thermal component of AGN is (with slight deviation) a power law [CO96]:

$$F_\nu(\nu) \propto \nu^{-\alpha},$$  \hspace{1cm} (1.3)

which is a strong indication of synchrotron radiation: This is energy released by electrons. The power law spectrum is the result of the integration over all single
electron spectra as shown in figure 1.6. However, the spectrum bends at a certain frequency $\nu \approx 5 \cdot 10^{12}$ Hz because the plasma becomes opaque to its own synchrotron radiation\(^8\). For values $\nu \leq 5 \cdot 10^{12}$ Hz, the spectral index is $\alpha < 2.0$. At frequencies $\nu > 5 \cdot 10^{12}$ Hz, the spectrum becomes as steep as $\alpha \approx 2.5$. This is only a simple ansatz for AGN spectra, since there are other interacting processes. From the spectra it can be concluded that there are obviously no atomic absorption and emission processes which means that the radiating material is completely ionized. Possible (non-atomic) processes are bremsstrahlung, inverse Compton scattering and electromagnetic cascades (induced by high energy $p - p$ or $\gamma - p$ interactions). But a power law spectrum is a good approximation because the synchrotron radiation dominates over all other radiation processes and the measured spectra are almost pure power law spectra. Figure 1.7 shows a sketch of a typical AGN spectrum. The turnover at $\nu \approx 5 \cdot 10^{12}$ Hz is, as mentioned previously, due to synchrotron self-absorption. The thermal component (blackbody radiation) is seen in the form of the blue bump at around $7 \cdot 10^{14}$ Hz $< \nu < 3 \cdot 10^{15}$ Hz. The UV bump is likely produced by processes in the disk. The radiation from the UV bump is also referred to as Eddington Radiation.

![Figure 1.7: Scheme of the continuum of a typical AGN spectrum [CO96].](image)

1.2.4 AGN classes

The different types of galaxies all classified as AGN because of their bright, compact core and their high luminosity in the radio are subdivided to the following smaller classes.

- *Seyfert* galaxies were the first objects observed with a very bright nucleus

---

\(^8\)This process is called *synchrotron self absorption*. 
almost stellar in appearance (Carl Seyfert, 1943). They are subdivided into galaxies with both broad \((v \approx 1000 - 5000 \text{ km/s})\) and narrow emission lines \((\text{Seyfert 1})\) and those with only narrow \((v \approx 500 \text{ km/s})\) emission lines \((\text{Seyfert 2})\). Seyfert 1 galaxies have very broad emission lines for allowed lines (such as H I, He I, He II) and narrower for forbidden lines (O III)\(^9\). There are a few spectra showing both broad and narrow allowed lines which are classified as \textit{Seyfert 1.5}. Seyferts are radio quiet compared to other types of AGN. The X-ray emission for Seyfert 1 and 1.5 is strong and very variable. The periods of one cycle vary between hours and days. On the other hand, X-ray emission for Seyfert 2 galaxies is not measured for most of these objects. If resolvable, Seyferts are found to be \textit{spiral galaxies} in at least 90% of all cases. Also, Seyferts are often accompanied by other galaxies, probably gravitationally interacting.

- **Quasars**: Radio telescopes in the late 1950s discovered strong radio sources with starlike appearances and unique spectra. These objects were later classified as quasi-stellar radio sources \((\text{quasars})\). They show emission lines which can only be identified with those in a resting frame by assuming a very high redshift\(^{10}\). Thus, quasars are the most distant galaxies detected so far. Most of the quasars are radio-loud, but there are a few radio quiet quasars, from now on called “Quasi Stellar Objects” - \textit{QSOs}. Further subclasses are \textit{SSRQs} (“Steep Spectrum Radio Quasars”), which have strong emission lines with extended radio emission and a steep spectrum \((\alpha > 0.5)\), and \textit{FSQR} (“Flat Spectrum Radio Quasars”) with compact radio emission and a flat spectrum \((\alpha < 0.5)\).

- **Radio Galaxies** are radio-loud galaxies which appear with narrow emission lines (Narrow Line Radio Galaxies, \textit{NLRG}) and with broad emission lines (Broad Line Radio Galaxies, \textit{BLRG}). Radio galaxies are (giant) ellipticals with extended radio emission and a steep spectrum \((\alpha \approx 0.8)\). A radio galaxy can appear with extended radio lobes or radiate its energy from a compact core and a halo which is about the size of a normal (visible) galaxy or larger.

- **Blazars** are distant galaxies which vary rapidly in brightness and which have a high degree of polarization. 90% of all resolved blazars are ellipticals. The most well-known object of this class is BL Lacertae in the northern constellation of Lacerta\(^{11}\). Objects with properties similar to BL Lacertae form a blazar subclass called BL Lac. BL Lac objects are characterized by their rapid time variability: In only 24 hours, their luminosities can change by as much as 30%. An explanation can be given in connection with the unified model which is described later. Furthermore, BL Lac spectra are highly polarized (that means 30 - 40% linearly polarized light) and the spectra barely show emission lines: Observations of the very rare and faint emission lines of BL Lac objects have shown that these lines are highly redshifted which means that BL Lacs are

\(^{9}\)Forbidden means a very low probability.

\(^{10}\)For example, quasar 3C 273 shows broad emission lines with the pattern of the Balmer series of hydrogen redshifted by \(z = 0.158\) which implies a velocity of \(v = 0.146 \cdot c\) [CO96]!

\(^{11}\)This is the Latin word for lizard.
found at cosmological distances. Another blazar-subclass are optically violent variable quasars (OVVs). They differ from BL Lacs by their higher luminosity and their broad emission lines.

Table 1.1 below shows different types of AGN classified by their radio activity, the width of the emission lines and the spectral index of their spectrum.

<table>
<thead>
<tr>
<th>Type of AGN</th>
<th>Radio</th>
<th>galaxy type</th>
<th>emission lines</th>
<th>spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seyfert 1</td>
<td>quiet</td>
<td>spiral</td>
<td>broad</td>
<td></td>
</tr>
<tr>
<td>Seyfert 2</td>
<td>quiet</td>
<td>spiral</td>
<td>narrow</td>
<td></td>
</tr>
<tr>
<td>Quasar QSO</td>
<td>loud</td>
<td>spiral</td>
<td>broad</td>
<td>flat (FSRQ) and steep (SSRQ)</td>
</tr>
<tr>
<td></td>
<td>quiet</td>
<td>elliptical</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blazar</td>
<td>loud</td>
<td>elliptical</td>
<td>narrow (BL Lac)</td>
<td>flat or inverse</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>broad (OVV)</td>
<td></td>
</tr>
<tr>
<td>Radio Galaxy</td>
<td>loud</td>
<td>elliptical</td>
<td>narrow (NLRG)</td>
<td>steep</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>broad (BLRG)</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.1: The AGN zoo

1.2.5 Morphology

Looking at the radio appearance of AGN, there are two morphological classes: Compact sources with a flat radio spectrum and extended sources with a steep spectrum. In this context, compact means that the emission region is restricted to a region which is much smaller than the typical diameter of a galaxy.

Extended sources vary strongly within their morphology: There are diffuse sources as well as structured sources with two or more luminosity centers. Furthermore, “double sources” are found, where two bright radio clouds are symmetrical to a less luminous core. These radio clouds are called knots.

In 1974, Fanaroff and Riley [FR74] classified extended radio sources by their morphology: Defining Q as the quotient between the distance of the luminosity centers\(^\text{12}\) and the total dimension of the source, Fanaroff and Riley divide AGN sources into two classes: They found a strong correlation between the value of Q and the absolute radio luminosity at a frequency of \(\nu = 0.178\) GHz and therefore classify AGN by their radio luminosity:

- **FR-I galaxies** are sources with a luminosity of \(F_{0.178} \leq 2.5 \cdot 10^{26}\) W/Hz (\(Q < 0.5\)). The radio luminosity decreases with the distance from the center. Knots and jets are observed in FR-I galaxies and the knots are located inside the galaxy (\(Q < 0.5\)). FR-I galaxies have a very complex structure and are often found in dense galaxy clusters.

- **FR-II galaxies** have an absolute radio luminosity of \(F_{0.178} \geq 2.5 \cdot 10^{26}\) W/Hz (\(Q > 0.5\)). The size of the luminosity centers range from 100 kpc up to

\(^{12}\)Without paying attention to the central source.
several Mpc. The jets and the knots appear much brighter in FR-II galaxies, which is why knots in FR-II galaxies are called hot spots. These hot spots are significantly brighter than the central source and in contrast to the knots in FR-I galaxies, FR-II hot spots are located outside the galaxy with a size of several kpc. FR-II galaxies appear mainly outside or at the edge of galaxy clusters.

Hot spots emit a broad electromagnetic spectrum, reaching from radio to optical emission. Since the emission is synchrotron radiation, diffuse shock acceleration can be assumed. This is the reason why AGN are likely to produce ultra high energy particles which contribute to the cosmic ray flux beyond the ankle.

1.2.6 Unification

The unified model predicts that all active galaxies classified and described previously are the same type of object. The appearance is different due to different orientations of the AGN as viewed from Earth and also because of different rates of accretion and masses of the central black hole. Figure 1.8 shows how the unification can be represented [ZB02]. Figure 1.9 shows an overlap of a radio (jet) and an optical image of NGC-4261. The magnification of the AGN core is an image taken with the Hubble Space Telescope.

All active galaxies seem to be driven by a central engine which is a rotating, supermassive black hole. Furthermore, matter is accreting around the black hole, presumably producing a magnetic field which can accelerate particles away from the galaxy. There are two preferential directions opposite to each other and perpendicular to the disk so that two relativistic particle jets can be observed. In the direction of the jets, there are minor gas clouds and radio lobes. Around the center there is an optically opaque torus and, closer to the center, there is the Broad Line Region. The unified model is a simple model and not all details concerning AGN can be explained by the unification so far, but there are some key pieces of evidence in support of it:

- Studies of the \( H_\alpha \)-line of different AGN [Shu81] show that the \( H_\alpha \) luminosity is proportional to the featureless continuum at 4800 Å (on a log-log plot, the slope is \( \sim 1.05 \)). This indicates a common origin for \( H \)-lines (both broad and narrow) in Seyfert 1, 2, BLRG, NLRG, quasars and QSOs.

- In 1985, R. Antonucci and J. Miller observed NGC 1068, a Seyfert 2 galaxy, in polarized light and found a Seyfert 1 spectrum with broad emission lines. This implies that the Seyfert 1 nucleus is hidden from the direct view of Earth by optically thick material: The Seyfert 1 spectrum is therefore usually diminished and overwhelmed by the Seyfert 2 spectrum.

- One hint for a central engine is the rapid time variability.
1.2. Active Galactic Nuclei (AGN)

![Diagram of AGN](image)

Figure 1.8: Scheme of a cylindrically symmetric AGN shown in the r-z-plane, both axes logarithmically scaled to 1 pc. It is indicated which objects are believed to be seen from which direction [ZB02].

1.2.7 Neutrino production in AGN

Neutrinos in AGN are believed to be produced via photo-meson production as explained in section 1.1.3. In the hadronic acceleration model, protons and neutrons interact with photons to produce mesons which again produce electron- and muon-neutrinos. The hot spots in the jet are moving with relativistic speed, having a Lorentz Boost factor of $\gamma = 10 - 100$. The nucleons are therefore accelerated via diffuse shock acceleration and interacting with the photons from the blue bump that originate in the disk. That is why ultra high energy particles from AGN are believed to be produced in the inner jet, in the vicinity of the disk (see figure 1.10). The big question is whether protons are produced in the jet (Hadronic model) or if the particle production in AGN is exclusively leptonic (Leptonic model). If there are no hadrons produced in the jet, no neutrinos will be produced by the AGN. This question is of special interest for the next generation of large volume neutrino detectors (see section 2).

1.2.8 Jet-disk symbiosis

There is strong evidence that AGN jets and disk are symbiotic features as it has been worked out by Falcke et al. [FMB95, FB95, Fal96]. The idea of the jet-disk symbiosis will be presented here along general lines. The model is based on observations of quasar luminosities at 5 GHz and it is developed for compact radio cores. If the
Figure 1.9: A radio image (jet), overlapping an optical image (disk) of the AGN NGC-4261 is shown on the right side. It has a width of 88000 ly and the two jets around a luminous core are noticeable. The right image has been taken by Hubble Space Telescope (HST) with a width of \( \approx 1250 \) ly. The torus around the core can clearly be seen.

The maximum accretion power of a quasar is \( Q_{\text{accr}} \), then the so called disk luminosity \( L_{\text{disk}} \), describing the blue bump luminosity, is directly correlated to \( Q_{\text{accr}} \):

\[
Q_{\text{accr}} = q_i \cdot L_{\text{disk}}
\]

as well as the total jet power \( Q_{\text{jet}} \):

\[
Q_{\text{jet}} = q_j \cdot Q_{\text{accr}}.
\]

Here, \( 5\% < q_i < 30\% \) and \( q_j < 1 \) are dimensionless fractions. The accretion power is given by the rate of change in the disk mass \( M_{\text{disk}} \):

\[
Q_{\text{accr}} = \dot{M}_{\text{disk}} c^2
\]

and therefore the mass that is swallowed by the black hole \( M_{\text{swallow}} \) is the total power minus the emitted power \( Q_{\text{jet}} \) and \( L_{\text{disk}} \):

\[
M_{\text{swallow}} = (1 - q_j - q_i) \dot{M}_{\text{disk}} c^2.
\]  

(1.4)

To determine the quasars radio luminosity, synchrotron radiation processes have to be considered. The synchrotron parameters needed here are taken from [RL79] as

\[
\epsilon_{\text{sync}} = 5.5 \cdot 10^{-19} \frac{\text{erg}}{\text{s} \cdot \text{cm}^2 \cdot \text{Hz}^{-1}} \left( \frac{B}{G} \right)^{3.5} \left( \frac{\nu}{\text{GHz}} \right)^{-0.5}
\]
Figure 1.10: Possible blueprint for the production of high energy neutrinos (and photons). Electrons and protons are accelerated in knots moving along the jet, interacting with photons from the disk [Hal98].

\[ \alpha_{\text{pitch}, \gamma} = 53.4^\circ \]
\[ \kappa_{\text{sync}} = 4.5 \times 10^{-12} \text{ cm}^{-1} \text{ s}^{-1} \]
\[ \alpha_{\text{pitch}, c} = 54.7^\circ . \]

Here, \( B/G \) is the magnetic field in units of Gauss and \( f \) describes the relative strength of the magnetic field energy to the relative electron density. Defining the gravitational radius \( R_g \)

\[ R_g = \frac{G \cdot M_{BH}}{c^2} \]

with \( M_{BH} \) as the mass of the black hole and \( G \) as the gravitational constant and the relative radial jet coordinate \( r_j \):

\[ r_j = \frac{R_{\text{jet}}}{R_g} \]

with \( R_{\text{jet}} \) as the radial jet coordinate, the optical depth \( \tau \) for electromagnetic radiation through the central axis of the jet is given by

\[ \tau = 2 r_j R_g \frac{r_{\text{syn}, c}}{\sin i} \]

with \( i \) as the inclination angle. Observing compact cores with an optical depth of \( \tau = 1 \), the part of the jet where synchrotron self absorption starts (for an observed frequency \( \nu_{\text{obs}} \)), \( Z_{\text{ssa}} \) is given by

\[ Z_{\text{ssa}} = 19 \text{ pc} \cdot \left( \frac{x_c^{1/2} \beta_j}{u_3 \cdot \gamma_{jj} \cdot f^{1/2} \cdot \sin i} \right)^{1/3} \frac{GHz}{\nu_{\text{obs}} / D} \left( \frac{q_j / L_{46}}{1} \right) \]

The parameters are explained in table 1.2. Thus, considering the vertical jet coor-
<table>
<thead>
<tr>
<th>parameter</th>
<th>explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_j,0 = 6 \pm 2$</td>
<td>Lorentz boost factor</td>
</tr>
<tr>
<td>$\beta_j,0 = 0.972 \pm 0.019$</td>
<td>velocity: $1 - \frac{1}{2\gamma_j,0^2}$</td>
</tr>
<tr>
<td>$i = \frac{1}{\gamma_j,0} = \frac{6}{6 \pm 18}$</td>
<td>inclination angle</td>
</tr>
<tr>
<td>$i_{\text{obs}} = \frac{1}{\gamma_j,0} = \frac{6}{6 \pm 18}$</td>
<td>angle between observer and jet axis</td>
</tr>
<tr>
<td>$L_{46} = \frac{L_{46}}{10^{46\text{erg}}}$</td>
<td>luminosity</td>
</tr>
<tr>
<td>$x_e\gamma_{\min,e} \approx 100$</td>
<td>$x_e$: rel. $e^-$ number density (units of total proton density)</td>
</tr>
<tr>
<td>$\gamma_{\min,e}$</td>
<td>minimal Lorentz factor of relativistic $e^-$ population</td>
</tr>
<tr>
<td>$D = \frac{1}{\beta_j,0} (1 - \beta_j,0 \cos (i_{\text{obs}}))^{-1}$</td>
<td>Doppler factor</td>
</tr>
<tr>
<td>$q_{j,1} = \frac{Q_{\text{jet}}}{L_{\text{disk}}} = 0.15^{+0.2}_{-0.1}$</td>
<td>total jet power in units: disk lum.</td>
</tr>
<tr>
<td>$\xi = 0.15^{+0.1}_{-0.15}$</td>
<td>fit parameter</td>
</tr>
<tr>
<td>$u_3$</td>
<td>ratio btw. tot. energy density in the jet and the magn. energy density, divided by a factor of 3</td>
</tr>
</tbody>
</table>

Table 1.2: Parameters needed for the jet-disk symbiosis model [Fal96]

dicate $Z_j$ normalized to the gravitational radius ($z_j := Z_j/R_g$), the total monochromatic luminosity per frequency interval can be written as

$$L_{\nu,s} = 4\pi \cdot \int_{Z_{sta}}^{\infty} \epsilon_{\text{sync}} \cdot \pi \cdot \gamma^2_j (z_j) \cdot dz_j.$$ (1.7)

To get an estimation of the observed fluxes, the quantities have to be transformed into the observer’s frame:

$$L_{\nu,\text{obs}}(\nu_{\text{obs}}) = D^2 \cdot L_{\nu}(\nu_{\text{obs}}/D)$$ (1.8)

$$\nu_s = \frac{\nu_{\text{obs}}}{D}$$ (1.9)

$$\sin i = D \sin i_{\text{obs}}.$$ (1.10)

The resulting luminosity per frequency interval is therefore

$$L_{\nu_{s,\text{obs}}} = 4.5 \cdot 10^{31} \frac{\text{erg}}{s \cdot Hz} \left(\frac{q_j/1L_{46}}{10^{46}}\right)^{1/2} \cdot D_{13/6} \sin i_{\text{obs}}^{1/6} \cdot \frac{f_{1/2}}{x_e^{11/12}} \cdot \frac{f_{1/2}}{\sqrt{\gamma_j,0^{11/6}}}.$$ (1.11)

The conclusions which can be drawn from the jet-disk model are

- Radio and disk are strongly correlated for quasars. That underlines the fact that jet and disk are symbiotic features.

- At a given luminosity, radio-loud quasar cores are brighter than those of radio-weak quasars.

- At disk luminosities of $L_{\text{disk}} \geq 10^{46}$ erg/s, radio-loud quasars can appear with a FR-II type jet. That means that quasars of luminosities $L_{\text{disk}} \geq 10^{46}$ erg/s can be identified with FR-II galaxies.
- The power of the jet is approximately $1/3$ of the disk luminosity:

$$Q_{\text{jet}} \approx \frac{1}{3} L_{\text{disk}}. \quad (1.12)$$

That means the jet is energetically important for disk structure and evolution.

- The model which has been developed for compact sources works for extended sources as well, it will be used in section 3.1.
1.3 Cosmology

When calculating the neutrino flux from very distant AGN it is necessary to consider cosmology. Although much progress has been made in determining the cosmological parameters, there are still big uncertainties in the measured values. This part tries to give a brief overview of cosmological models and their parameters.

1.3.1 Cosmological parameters

The Hubble constant $H(t)$ is given by

$$H(t) = \frac{\dot{R}(t)}{R(t)}.$$  \hspace{1cm} (1.13)

Here, $R(t)$ is the scale factor. The Hubble constant is a measure of the expansion per time interval of the Universe. For historical reasons, $H$ is called a constant, but today there are implications that $H$ is varying with time. The Hubble constant observed today, $H_0$, can be expressed as [Wei72]:

$$H_0 = 10^5 h \cdot \frac{km}{s} \frac{1}{Gpc},$$  \hspace{1cm} (1.14)

where $h$ is the dimensionless Hubble parameter. Measurements of the CMB [B+03] restrict the parameter to:

$$h = 0.71^{+0.04}_{-0.03}$$  \hspace{1cm} (1.15)

In the following calculations, $h$ will be chosen to be 0.71. The big problem in determining the Hubble constant is that distances are not easily measured.

The deceleration parameter $q(t)$ is the change in the expansion rate, i.e. if the Universe is decelerating $q > 0$ or if it is accelerating $q < 0$:

$$q(t) = -\frac{\ddot{R}(t)R(t)}{R(t)^2}.$$ \hspace{1cm} (1.16)

The normalized matter density $\Omega_m$ is set to

$$\Omega_m(t) = \frac{8\pi G}{3H(t)^2} \rho(t).$$  \hspace{1cm} (1.17)

If $\Omega_m = 1$, the mass density is critical ($\rho = \rho_c = 3H(t)^2/8\pi G$) in the sense that in a flat Universe ($k = 0$) with $\Lambda = 0$, $\rho = \rho_c$ gives the critical value between collapse and eternal expansion.

The normalized cosmological constant $\Omega_\Lambda$ is defined analogously to the normalized matter density:

$$\Omega_\Lambda(t) = \frac{c^2}{3H(t)^2} \Lambda.$$ \hspace{1cm} (1.18)
1.3. Cosmology

The critical cosmological constant, \( \Lambda_c \), is defined as the constant which Einstein tried to use to keep the Universe static. A static Universe implies \( R = \dot{R} = 0 \) which, leads to\(^{13}\)

\[
\Lambda_c = 4 \pi G \rho(t) c^2 .
\] (1.19)

This state is, however, unstable so that Einstein finally condemned the idea of a cosmological constant.

The normalized curvature \( \Omega_k \) is defined as

\[
\Omega_k = -\frac{c^2 k}{R(t)^2}.
\] (1.20)

The state of the Universe is totally determined by the previously mentioned parameters, since the Einstein equations can now be written as

\[
\Omega_\Lambda(t) - \frac{1}{2} \Omega_m(t) = -q(t) \quad (1.21)
\]

\[
\Omega_m(t) + \Omega_\Lambda(t) + \Omega_k = 1 .
\] (1.22)

The fundamental cosmological diagram (figure 1.15) shows \( \Omega_\Lambda \) vers. \( \Omega_m \) and the critical thresholds for the geometry of the Universe as well as expansion and collapse. Several models have been presented in the past (i.e. the Einstein-de Sitter model with \( k = 0, \Lambda = 0, q = 1/2 \) which would lead to an eternal expansion). Today, it is well determined that there is a non-zero cosmological constant (\( \Omega_\Lambda = 0.75 \pm 0.04 \)) and a material density \( \Omega_m = 0.27 \pm 0.04 \) [S\(^+\)03a].

1.3.2 Cosmological distances

Looking at an object at a distance \( r_1 \), neither the distance \( r_1 \) nor the lookback time \( t_1 \) is directly measurable. Instead, measurable quantities are introduced.

The angular distance is defined as

\[
d_A = \frac{D}{\theta}
\] (1.23)

where \( D \) is the known proper size of the object and \( \theta \) is the angular diameter. The proper motion distance is given by the known velocity \( u \) of an object per apparent angular motion \( \theta \). The luminosity distance \( d_L \) is defined as [CO96]

\[
d_L^2 = \frac{L}{4 \pi F}.
\] (1.24)

Here, \( F \) is the radiant flux measured for a light emitting source with a luminosity \( L \). The relation between these measurable quantities and theoretical quantities such

\(^{13}\) Using the Friedmann equations.
as the scale factor at present time \(R_0\) and at the time \(t_1\) at which the object is seen \(R_1\) is \([\text{RL79}]\)

\[
(1 + z) = \frac{R_0}{R_1} \quad d_A = R_1 \cdot r_1 \quad (1.25)
\]

\[
d_M = R_0 r_1 \quad d_L = \frac{R_0^2 r_1}{R_1}. \quad (1.26)
\]

Thus, the distance measures are connected by the redshift:

\[
d_L = (1 + z) \cdot d_M = (1 + z)^2 \cdot d_A. \quad (1.27)
\]

Using the integral presentation of the lookback time (see \([\text{CPL92}]\)), the luminosity distance can be written as a function of \(z_1\):

\[
d_L = \frac{(1 + z) \cdot c}{H_0 \cdot |\Omega_k|^{1/2}} \sinh \left\{ \left| \Omega_k \right|^{1/2} \cdot I(z) \right\}. \quad (1.28)
\]

Here, “\(\sinh\)” is defined as follows \([\text{CPL92}]\):

\[
sinh = \begin{cases} 
\sinh & \text{for } \Omega_k > 0 \\
\sin & \text{for } \Omega_k < 0 \\
\text{disappears together with both } \Omega_k & \text{for } \Omega_k = 0
\end{cases}. \quad (1.29)
\]

and \(I(z)\) is an integral

\[
I(z) := \int_0^z \left[(1 + z')^2 \cdot (1 + \Omega_m z') - \Omega_A z' (2 + z') \right]^{-1/2} dz'. \quad (1.30)
\]

The integral can be evaluated numerically with the result shown in figure 1.11. In this figure, the three distance measures are compared using the current values for \(\Omega_m\) and \(\Omega_A\) (with an approximation that \(\Omega_k \approx 0\)).

### 1.3.3 The Comoving Volume

When counting objects at cosmological distances, it is common to determine the comoving densities of the objects. To estimate the actual number of galaxies, these densities have to be multiplied with the comoving volume.

The comoving volume element in the Robertson-Walker metric is given by \([\text{CPL92}]\)

\[
dV_c = \frac{d^2 M}{(1 + \Omega_k \cdot (H_0/c)^2 \cdot d_M^2)^{1/2} \cdot d(d_M) d\Omega}. \quad (1.31)
\]

The integral over the solid angle and the motion distance can be evaluated analytically with the result

\[
V_c(d_M) = \begin{cases} 
\left( \frac{H_0}{H_0} \right)^3 (2\Omega_k)^{-1} \left[ H_0/c \cdot d_M (1 + \Omega_k (H_0/c)^2 d_M^2)^{1/2} \right. \\
\left. - \left| \Omega_k \right|^{-1/2} \sin^{-1} (H_0/c \cdot d_M |\Omega_k|^{1/2}) \right] & \text{for } \Omega_k \neq 0 \\
\frac{4\pi}{3} d_M^3 & \text{for } \Omega_k = 0
\end{cases} \quad (1.32)
\]
1.3. Cosmology

Figure 1.11: Distance measures versus redshift for $\Omega_m = 0.27$ and $\Omega_\Lambda = 0.75$, taking $\Omega_k \approx 0$ as assumed in [S+03a].

1.3.4 Cosmic microwave background

In 1964, Penzias and Wilson discovered the cosmic microwave background (CMB) [HH98] with a sensitive radio antenna which was originally used to study radio emission from the Galaxy. Today it is known that the CMB is thermal (blackbody) radiation, distributed isotropically in space with a temperature of $T = 2.735$ K [M+90]. This isotropic radiation goes back to approximately $10^{-6}$ seconds after the Big Bang, when quarks condensed into hadrons. All baryon-antibaryon pairs annihilated to leave behind photons: These photons make up most of the cosmic microwave background today. When the temperature of the Universe had dropped to $T \approx 3000$ K, the free electrons did not have sufficiently high energy to escape the electric field of the protons and thus they were captured by primordial nuclei. This phase is called recombination. Before recombination photons were tightly coupled to the freely moving electrons through Compton scattering. Due to the coupling of the electrons to the primordial nuclei the photons could escape and the Universe became transparent. That implies that the observable Universe begins at $T \approx 3000$ K which corresponds to $z \approx 1100$ and a time $t \approx 10^5$ years after the Big Bang [Dur01].

To confirm that the CMB spectrum is truly isotropic, very accurate measurements are necessary: The early ground-based measurements still allowed a small anisotropy since the blackbody spectrum was disturbed by fluctuations in the atmosphere. In 1989, the Cosmic Background Explorer (COBE) satellite was launched. COBE was able to measure the intensity of the CMB without interference from the atmosphere. The result is shown in figure 1.12 with the best fit for a blackbody with a temperature
of 2.735 K. Although the spectrum is nearly isotropic, a dipole irregularity occurs
due to the motion of Earth through the Universe\textsuperscript{14}. Using COBE data it was possible to
determine the Earth’s velocity around the sun (29 km/s), the sun’s local velocity in the milky way (220 km/s) and the
speed of our galaxy in the local cluster (600 km/s). Subtracting this effect as well as the radiation from the galactic
disk anisotropies on the $\mu$K scale remain which are due to density fluctuations at the
surface of last scattering. These density fluctuations can be traced back to a time at $z \approx 1500$ and conclusions about the very early Universe are possible.

1.3.5 Measuring cosmological parameters

Two different methods are used to determine the current cosmological parameters,
namely the small scale anisotropy measurements of the CMB [Dur01] and very early
type Ia supernovae [PS97].

To measure the small scale anisotropies of the CMB, the temperature fluctuations are
expanded into spherical harmonics,

$$T(\theta, \phi) = \sum_{l,m} a_{l,m} Y_{l}^{m}(\theta, \phi) .$$  \hspace{1cm} (1.33)

Here, $a_{l,m}$ are the expansion coefficients and $Y_{l}^{m}$ are the spherical harmonics:

$$Y_{l}^{m} = (-1)^{m} \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_{l}^{m}(\cos \theta) e^{im\phi} ,$$  \hspace{1cm} (1.34)

where $P_{l}^{m}(\cos \theta)$ are the associated Legendre polynomials. The amplitude of the
coefficients is defined as the mean value in $m$ of the absolute square of $a_{l,m}$:

$$C_{l} := \left< |a_{l,m}|^{2} \right>_{m} .$$  \hspace{1cm} (1.35)

\textsuperscript{14}The fluctuations are around a temperature scale of $\Delta T \approx \text{mK}$.
The angular power spectrum is then represented by \( l(l+1) \cdot C_l \), where \( l \) is the multipole order. Figure 1.13 shows the current experimental result measured up to \( l \approx 1500 \). The spectrum can be evaluated theoretically and fitted to the data by varying the cosmological parameters. The best fit parameters are given in table 1.3.

![Angular scale in degrees](image)

**Figure 1.13:** The measured CMB spectrum taken from [TZ02]. Measurements reach up to \( l \approx 1500 \). Data are combined from all available experiments.

<table>
<thead>
<tr>
<th>( \Omega_b h^2 )</th>
<th>( \Omega_\Lambda )</th>
<th>( \Omega_m )</th>
<th>( \Omega_{tot} )</th>
<th>( h )</th>
<th>Age (Gyr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.022^{+0.009}_{-0.009}</td>
<td>0.75^{+0.04}_{-0.04}</td>
<td>0.27^{+0.02}_{-0.02}</td>
<td>1.02^{+0.02}_{-0.02}</td>
<td>0.71^{+0.04}_{-0.03}</td>
<td>13.7^{+0.2}_{-0.2}</td>
</tr>
</tbody>
</table>

**Table 1.3:** Current cosmological parameters [S+03a]. Here, \( \Omega_b \) is the baryonic matter density. The results are mean and with 68% confidence errors. Results are fit to the WMAP, CBI, ACBAR, 2dFGRS and Lyman \( \alpha \) forest data.

Another method to determine the cosmological parameters (\( \Omega_m, \Omega_\Lambda \) and \( h \)) is using distant supernovae (SNe). Since the type Ia SN can be used as a standard candle, the conversion from luminosity to redshift is possible. The relation between effective magnitude and redshift can be measured and the data can be fitted with different cosmological parameters which is shown in figure 1.14. The constraints to the cosmological parameters \( \{ \Omega_m, \Omega_\Lambda \} \) are shown in figure 1.15 where the best-fit confidence regions in the \( \Omega_m - \Omega_\Lambda \) plane are shown (68\%, 90\%, 95\% and 99\%). The “Supernova Cosmology Project” [PS99] has taken these data using different telescopes such as the Hubble Space Telescope, Keck and others to observe the sky shortly after the new moon. About three weeks later, the same regions in the sky are observed again in order to compare the pictures. If there is a relevant luminosity peak, this object is likely to be a supernova and the light curve can be taken while still brightening since an average SN has a rise time > 3 weeks.
Figure 1.14: Stretch corrected Hubble plot for SN-Ia. The boxed plot shows high redshifted supernovae where the deviation from the Einstein-de-Sitter model (EdS, \( \{k = 0, \Omega_A = 0, \Omega_m = 0\} \)) is significant [S+03b].
Figure 1.15: The cosmological diagram taken from [PS99]. The two lines determine whether the Universe is flat, closed or open and if it will expand eternally or recollapse. Combining the SN data with the CMB data, the Universe is flat ($\Omega_m + \Omega_\Lambda = 1$). A $\Lambda = 0$ Universe can be ruled out by SN measurements with a confidence of $P(\Lambda > 0) = 99\%$. The upper shaded region represents cosmologies which do not allow a big bang (here, the age of the Universe diverges) and the lower shaded region corresponds to a Universe that is younger than the oldest heavy elements.
Chapter 2

Experiments

In this chapter, the main cosmic neutrino telescope techniques as well as current and future experiments will be presented. All detectors use large arrays of (more or less) dense matter (water, ice, salt or air) to observe muons which are produced when cosmic neutrinos interact with protons or neutrons. Since the cross section for such an interaction is very small large arrays are needed to get a significant neutrino rate. Current experiments measure the atmospheric neutrino spectrum while future experiments hopefully are able to detect the extraterrestrial neutrino flux component.

2.1 ... in water and ice

Muon-neutrinos traveling through water or ice can interact with the medium or neutrons and produce muons. In a water-like medium, the muon travels faster than light in the same medium and is therefore producing a Cherenkov cone. The Cherenkov light emitted by the muon can be detected by an array of Optical Modules (OMs) consisting of Photo Multiplier Tubes (PMTs) embedded in pressure resistant glass shells. The spacing of the PMTs is determined by the absorption length of the Cherenkov light in the medium. The size of the array is chosen to be as large as possible since the neutrino flux decreases with energy. Thus, the detection rate of (ultra) high-energy events increases with the detector size. Another problem is how to distinguish between neutrino-induced muons and atmospheric muons from CR interactions. This is solved by using the Earth as a filter: Down-going events have a very large rate of CR muons\(^1\) while up-going events can be considered to be neutrino-induced muons since all CR muons will decay while transversing the Earth. Thus, using large natural water or ice volumes is a very efficient method to measure the neutrino flux.

The principle of a neutrino detector in a water-like medium can be explained with the example of the AMANDA (Antarctic Muon and Neutrino Detector Array) experiment which is located near the geographical South Pole at approximately 1500–2000 m depth in the ice. This depth is primarily necessary in order to reduce the back-

\(^1\)The ratio between raw triggered muons and neutrino-induced muons in AMANDA is \(10^6 : 500\)
ground rate of down-going muons and secondly, the ice can be considered to be without bubbles or other disturbing features so that the Cherenkov light travels in straight lines without being refracted. The AMANDA-A detector was built at a depth of approximately 600 – 800 m. The ice is not bubbleless at this depth so that the successors were placed deeper in the ice. The AMANDA-B10 detector completed in January 1997 consists of 302 optical modules (OMs) on ten strings. The strings are arranged cylindrically with a diameter of 120 m and a height of 500 m. The AMANDA-B10 array was enlarged in January 2000 to the AMANDA-II detector with 677 OMs on 19 strings. A model of the AMANDA detector is shown in figure 2.1. There are current AMANDA limits on the atmospheric neutrino flux up to energies of 100 TeV [Gee03]. The future project, IC3CUBE is an enlargement of the AMANDA detector to a 1 km$^3$ neutrino detector with 4800 OMs on 80 strings. A volume of 1 km$^3$ is promising to give predictions on the extraterrestrial neutrino flux, especially on the extragalactic component such as the AGN or GRB flux. Predictions about the neutrino event rate from AGN or other extragalactic objects (GRB, TD, etc.) are therefore quite important.

Other experiments are being built in water, such as Baikal, NESTOR and ANTARES [DB02, BN98, Mon03]. The Baikal experiment is located in the Baikal lake, 3.6 km away from the shore at 1.1 km depth [DB02]. The detector has an effective area of approximately $\sim 10^3 - 5 \times 10^3$ m$^2$ and has an umbrella like frame. On each of the eight strings 24 pairs of PMTs are mounted. Two PMTs in a pair are switched in coincidence to suppress the bioluminescence and the PMT noise. The

Figure 2.1: Sketch of the AMANDA detector. Left: the Eiffel tower for comparison. Middle: the AMANDA-B10 detector, enlarged. Right: a detailed sketch of an optical module (OM) [CA02].
OMs in the Baikal experiment have to lie close to each other since the absorption length of Cherenkov light is only 8 m.\textsuperscript{2} This also has advantages since the low energy threshold is very low so that it should for instance be possible to detect WIMPs.\textsuperscript{3} The ANTARES project (Astronomy with a Neutrino Telescope and Abyss environmental RESearch) started in 1996 and will be located at a depth of 2400 m in the Mediterranean Sea, 37 km off-shore of La Seyne sur Mer which is near Toulon (France). It has a surface area of 0.1 km\textsuperscript{2} and is supposed to be completed by the end of 2004 [Mon03]. Once complete, ANTARES will consist of ten strings with totally 1000 PMTs. NESTOR (Neutrino Extended Submarine Telescope with Oceanographic Research) is also an underwater neutrino detector which is located in the Ionian Sea south west of Peloponnesos (Greece) [BN98]. The detector is built in 3800 m depth, 13 nautical miles from the coast. The first phase of the detector consists of a hexagonal tower with 168 OMs. The second step foresees the deployment of six more towers around the first one with a distance of 140 m between the towers and a total of 1176 PMTs. The final goal is to reach an array of 13 towers and 24 strings with totally 2760 OMs and a surface of of 1 km\textsuperscript{2}. One tower consists of twelve hexagonal semirigid floors with a spacing of 30 m in between the floors. The PMTs are paired so that one PMT looks up and the other down. This leads to a total up-down symmetry. In 2002, the first tower has been completed and the next towers are being built this year.

An alternative detection method is the observation of Cherenkov light at radio wavelength with RICE (Radio Ice Cherenkov Experiment) which is connected to AMANDA. The detection method will be described more closely in section 2.3 since it can easily be used for detectors in large salt mines. The RICE experiment was initiated in 1995 by the AMANDA collaboration which consented to co-deployment of two dipole radio receivers in the first holes for the AMANDA strings [K+06]. Today, the RICE experiment consists of a 16-channel array of dipole receivers within a volume of (200 m)\textsuperscript{3} [K+03].

2.2 Airshower arrays

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.2}
\caption{Scheme of a horizontal tau neutrino airshower [Cro01].}
\end{figure}

\textsuperscript{2}In comparison, the absorption length in the antarctic ice is 100 m.

\textsuperscript{3}WIMPs are \textit{Weakly Interacting Massive Particles} which are assumed to contribute to dark matter. These particles are very massive so that they assemble in gravity centers such as center of the Earth or the sun. Heavy supersymmetric particles are WIMP candidates for instance.
The principle of neutrino detection with airshower arrays can be illustrated with the example of the Pierre Auger Observatory, located in the Pampa Amarilla in Argentina, an area with the advantage that it is very flat and dry. The sky is very clear due to the dry air so that observations with a fluorescence detector array are possible. A surface detector (SD) and a fluorescence detector array (FDA) cover a detection area of 3000 km². The SD will consist of 1600 watertanks, each 10 m² in area and 1.2 m in depth. The spacing between the water tanks is 1.5 km. The Cherenkov light in the water tanks is detected by three PMTs, planned to be placed directly beneath the cover of the tank looking downward. The FDA consists of 24 telescope units detecting fluorescence light from the airshowers. The two detection methods complement one another: The SD gives the lateral distribution of particles and opens the possibility to classify the primary hadron. The FDA is able to reconstruct the longitudinal development of the shower so that the interaction height of the primary particle and thus the energy and the track can be concluded.

Neutrinos can be detected by looking at horizontal air showers [Cro01]. Nearly horizontal cosmic rays have a high interaction probability. For an incoming particle with a zenith angle of 85° and an energy of \( E \approx 10^{19} \) eV, the height of conversion is around 40 km. The mean free path is very long so that most of the pions have the time to decay into muons. This results in a bundle of penetrating muons which are likely to reach the ground before decaying. Looking at neutrinos which interact at distances smaller than 3 km above the surface when having a zenith angle larger than 75°, muonic and electromagnetic components can be observed. The characteristics are the same as that of a vertical shower, but turned on its side. The showers initiated by a neutrino in the deep atmosphere can therefore easily be distinguished from the showers initiated in the upper atmosphere: Neutrino-induced showers will still contain electromagnetic energy while the showers from the upper atmosphere purely consist of muons and muon bremsstrahlung. The difference is seen in the time spread of the arriving particles in the detector: Upper atmosphere induced showers have a time duration of < 100 ns while neutrino-induced showers last several µs. Taking a very conservative point of view by assuming that the events above \( 10^{20} \) eV are exclusively produced by TDs, Auger expects to detect six events per year [Aug98].

### 2.3 ... in salt

A theory of Cherenkov radiation at radio frequencies was developed in the 1960s by G. A. Askaryan [Ask62]. An electromagnetic bunch is caused by a neutrino and knocks out electrons from the molecules in the atmosphere due to Compton scattering. Positrons from pair creation annihilate with electrons in the atoms which leads to a negative charge excess of approximately 30%. A bunch of these charged particles has a diameter of \( \sim 10 \) cm [Sal]. Looking at the power \( P \) of Cherenkov radiation, it is proportional to frequency \( \nu \) and bandwidth \( \Delta \nu \):

\[
P \propto \nu \cdot \Delta \nu.
\]

The index of refraction for light in a medium changes back to \( n \approx 1 \) at frequencies above the UV light so that the differential power decreases. Therefore, Cherenkov
radiation is optically seen as blue light. Since the power is proportional to the squared electrical field,

\[ P \sim |\vec{E}|^2 \]

and for optical frequencies, \(|\vec{E}| \sim \sqrt{N}\) with N as the number of particles, the power is proportional to N:

\[ P_{\text{opt}} \propto N. \]

For lower frequencies the corresponding wavelength is bigger than the size of the bunch (\(\lambda \geq d\)) and the electromagnetic fields of the particles are in phase \([S^+01]\) so that \(|\vec{E}| \propto N\) and

\[ P_{\text{rad}} \propto |\vec{E}|^2 \propto N^2. \quad (2.1) \]

Therefore, the ratio between radio power \(P_{\text{rad}}\) and optical power \(P_{\text{opt}}\) of the Cherenkov light is

\[ \frac{P_{\text{rad}}}{P_{\text{opt}}} = N \left(\frac{\nu_{\text{rad}}}{\nu_{\text{opt}}}\right)^2. \quad (2.2) \]

Typical frequencies are \(\nu_{\text{rad}} = (100 - 500)\) MHz (corresponding to wavelength \(\lambda_{\text{rad}} = (3 - 0.6)\) m) and \(\nu_{\text{opt}} = 75 \cdot 10^6\) MHz (optical blue light, \(\lambda_{\text{opt}} = 400\) nm). Thus, the ratio between optical and radio power is

\[ \frac{P_{\text{rad}}}{P_{\text{opt}}} \approx (1.8 - 44.4) \cdot 10^{-20} N. \quad (2.3) \]

Although a particle excess of \(N \approx 10^{18} - 10^{20}\) would be necessary to reach an intensity equivalent to the optical intensity\(^4\), recent experiments \([S^+01]\) show that such high intensities are not needed for a detectable Cherenkov signal. Therefore, the detection of the radio signal of Cherenkov radiation is an alternative method which is already used by the RICE experiment, located in the Antarctic ice (see section 2.1). Another approach is using large salt mines as a natural detector for Cherenkov light. Optical Cherenkov radiation is obviously absorbed while the radio signal would be detectable by helical radio antennas \([\text{Bec02}]\). The project is quite promising since salt has a density of \(\rho_{\text{salt}} = 2.2\rho_{\text{water}}\) where \(\rho_{\text{water}}\) is the water density so that the interaction probability in a 1\(km^3\) salt detector is higher than for a 1\(km^3\) water/ice detector. Furthermore, there are tests going on in US salt mines about the properties of the salt which show that the radio Cherenkov radiation does not bend significantly on salt layers or impurities. Although a proposal for a certain experiment has not been made yet, a project - in literature referred to as SALSA (SALtbed Shower Array) - is thought through and is hopefully proposed in the near future.

\(^4\)Which would imply an incident particle energy of \(\sim 10^{26}\) eV!
Chapter 3

A model for the neutrino energy spectrum from quasars

In the following, a detailed description of the calculation for a model of the isotropic neutrino flux from Active Galactic Nuclei (AGN) is given. The model only includes FR-II galaxies, sources with strongly extended radio emission. The neutrinos are assumed to be produced in \( p - \gamma \) interactions where mesons are produced decaying into leptons. The high-energy protons come from the inner part of the jet and interact with photons from the disk. The chapter is organized as follows: The first section comprises an explicit description of the models which are used in the calculation and gives an overview over the applied normalizations and the fits that have been made. In the second part, these models are used to calculate the quasar neutrino flux. The results of the calculation are discussed. Finally, an outlook on further work will be given.

3.1 Ingredients

The isotropic neutrino flux depends on two distributions:

1. The generic AGN neutrino energy spectrum, \( dN/dE_\nu \).

2. The AGN space density \( n(z, L) \) which is a function of luminosity and redshift.
   This is a convolution of the luminosity dependent AGN density normalized in \( z \) to \( z = 0 \), \( dn/dL \), and the AGN distance distribution (depending only on \( z \): \( \Psi(z) \)) normalized to \( \Psi(z = 0) = 1 \). This implies the approximation that the luminosity function is independent of the redshift \( z \).

In order to calculate the total neutrino flux it is necessary to fold these two distributions.

In this calculation, it has to be considered that the flux decreases with cosmological distance so that a factor \( 1/(4\pi d_L^2) \) has to be applied where \( d_L \) is the luminosity distance. This product has to be multiplied by the comoving volume \( dV_c/dz \). The result is a flux at a given redshift and a given luminosity. To obtain the flux which is
arriving at Earth ($\Phi(E^0_\nu)$, with $E^0_\nu$ as the neutrino energy at Earth), the expression has to be integrated over redshift and luminosity:

$$\Phi(E^0_\nu) = \int_z \int_L dzdL \frac{dN}{dE_\nu} \cdot \frac{dn}{dL} \cdot \Psi \cdot \frac{dV_c}{dz} \cdot \frac{1}{4\pi d^2_L}. \quad (3.1)$$

The integration over the luminosity has to be performed before the redshift integration, since the lower luminosity limit implicitly depends on the redshift (see also section 3.1.4). The result is the neutrino flux per steradian, area, second and energy ([\Phi] = 1/(sr \cdot s \cdot cm^2 \cdot GeV)). The functions which are mentioned above will be explained and normalized in this section.

PARAMETERS AND VARIABLES

To simplify the understanding of the calculation, a list of parameters is given below.

- $P_{rad}$: Radio power, here taken at the frequency $\nu = 0.5$ GHz in units of [erg/s/Hz].
- $P_{obs}$: Radio power, taken at the frequency $\nu = 5$ GHz, for extended quasars.
- $L_{radio}$: Radio luminosity in units of erg/s. This is the radio power, integrated over the frequency.
- $L_{disk}$: The luminosity of the blue bump. It is denoted $L_{disk}$, since the blue bump is in most cases produced by photons from the accretion disk.
- $L_{39}$: Disk (blue bump) luminosity in units of $10^{39}$ erg/s: $L_{39} = L_{disk}/(10^{39}$ erg/s).
- $E_\nu$: Neutrino energy at the AGN.
- $E^0_\nu$: A neutrino of the energy $E_\nu$ at an AGN has a remaining energy $E^0_\nu = E_\nu/(1 + z)$ at the Earth (due to cosmological expansion).
- $H_0$: Hubble parameter today, $H_0 = H(t = t_0)$.
- $h$: The Hubble parameter in units of 100 km/(s \cdot Mpc).
- $c$: The speed of light in vacuum.
- $\Omega_\Lambda$: The vacuum energy density due to the cosmological parameter $\Lambda$.
- $\Omega_m$: The matter density normalized to the critical density which gives the critical value between collapse or eternal expansion of the Universe in a $\Lambda = 0$ Universe.
- $\Omega_k$: The normalized curvature (see section 1.3).
- $d_L$: Luminosity distance.
NOTE

- The cosmological parameters are today experimentally well determined (see [S+03a] and [PS97]). Inflationary theory implies that the Universe is flat ($\Omega_k = 0$). The matter density of the Universe will be set to $\Omega_m = 0.27^{+0.04}_{-0.02}$ while the energy density is $\Omega_A = 0.73^{+0.06}_{-0.04}$ in the following. The Hubble parameter is set to $h = 0.71^{+0.04}_{-0.03}$ [S+03a]. The matter density comprises all matter, baryonic and non-baryonic.

3.1.1 The generic neutrino flux

The generic AGN neutrino flux is adopted to follow a power law with a spectral index of $\gamma = 2$:

$$\frac{dN}{dE_\nu} = \phi_\nu (L_{\text{disk}}) \cdot E_\nu^{-\gamma},$$

under the condition $E_\nu < E_{\nu_{\text{max}}} (L_{\text{disk}})$. Here, $E_\nu$ is the neutrino energy at the AGN. The generic spectral index of $\gamma = 2$ has been derived in e.g. [MPR01]. The normalization $\phi_\nu$ can be found by assuming that particles from an AGN produce the same order of magnitude of luminosity as the electromagnetic spectrum, $L_{\text{disk}}$, and neutrinos contribute with a fraction $x < 1$ [FMB95]. This is under the assumption that the power of the neutrinos is directly connected to the power of the jet which is proportional to the disk luminosity which in turn is a major fraction of the accretion power at high accretion rate. The factor $x$ is assumed to be in the range of $x = 1/30 \ldots 1/3$ [Fal].

$$\int_{E_{\nu_{\text{min}}}^\nu}^{E_{\nu_{\text{max}}}^\nu} E_\nu \cdot \frac{dN}{dE_\nu} \cdot dE_\nu = \int_{E_{\nu_{\text{min}}}^\nu}^{E_{\nu_{\text{max}}}^\nu} E_\nu \cdot \phi_\nu \cdot E_\nu^{-\gamma} dE_\nu = x \cdot L_{\text{disk}}.$$  

(3.3)

Using $\gamma = 2$ and expressing the luminosity in units of $10^{39}$ erg/s this is:

$$\int_{E_{\nu_{\text{min}}}^\nu}^{E_{\nu_{\text{max}}}^\nu} \phi_\nu \cdot E_\nu^{-1} dE_\nu = x \cdot 10^{39} \cdot L_{39} \left[ \frac{\text{erg}}{s} \right],$$

(3.4)

$$\ln \frac{E_{\nu_{\text{max}}}^\nu}{E_{\nu_{\text{min}}}^\nu} \phi_\nu = x \cdot 10^{39} \cdot L_{39} \left[ \frac{\text{erg}}{s} \right].$$

(3.5)

The normalization constant for the generic spectrum can be determined in units of $[\phi_\nu] = \text{GeV/s}$: Using that the minimum energy for meson production in $p - \gamma$ interactions is the rest mass of the charged pion, $E_{\nu_{\text{min}}} = E_{\pi^\pm} = 139.57$ MeV, and considering the maximum neutrino energy to be
d $E_{\nu_{\text{max}}} = (10^{10.51} \pm 10^{10.23})$ GeV the logarithm above can be regarded as a constant: $\ln \left( E_{\nu_{\text{max}}}^\nu/E_{\nu_{\text{min}}}^\nu \right) = 12.0 \pm 0.5$. Therefore, the normalization constant is

$$\phi_\nu = \chi \cdot 10^{39} \cdot L_{39} \left[ \text{GeV/s} \right],$$

(3.6)

where $\chi$ is in the range of $0.144 < \chi < 8.594$ and the generic spectrum is given by

$$\frac{dN}{dE_\nu} = \chi \cdot 10^{39} \cdot L_{39} \cdot E_\nu^{-2} \left[ \frac{1}{\text{GeV} \cdot s} \right].$$

(3.7)

\[1\] The maximum neutrino energy varies with the luminosity. This fact is discussed in detail in section 3.1.4.
Figure 3.1: Radio luminosity function (RLF) at 1.4 GHz taken from Windhorst (1984). For this investigation the sample of quasars was used [Win84].
3.1.2 The AGN density

The AGN density depends on the luminosity and on the redshift. Here, a density which depends only on the luminosity while the redshift evolution is factored off. The luminosity function is therefore valid at $z = 0$. Multiplying this function with the AGN distance distribution $\Psi$, depending only on the redshift and normalized to $\Psi(z = 0) = 1$, gives the density distribution of AGN.

The quasar luminosity function

Radio luminosity functions (RLFs) A compilation of the integral radio galaxy luminosity functions $(n(L, z))$ is given in [Win84], at a frequency of 1.4 GHz. The radio luminosity function (RLF) of different galaxy types is shown in figure 3.1. For the present analysis, the quasar line is of special interest: Radio quasars are obviously the most luminous objects and the radio luminosity is believed to be directly connected to the proton/neutrino production rate. The quasar line in figure 3.1 is an approximation based on calculations made by Schmidt et al. in [Sch72]. The original function from [Sch72] is shown in figure 3.3. The sample is provided by sources in the Revised Third Cambridge Catalogue (3CR) and was measured at a frequency of 0.5 GHz. According to Schmidt et al., the spectral index is $\alpha_{\text{spect}} \leq 0.5$ for about 90% of all quasars at 0.5 GHz. In further calculations, a spectral index of $\alpha_{\text{spect}} = 0.5$ as a lower limit is used. To determine the luminosity function, the observed quasar luminosities at a given redshift $z$ have to be normalized to $z = 0$ so that a distance independent function is achieved. To do so, a space evolution function has to be applied to the observed data. Two different evolutions have been considered by Schmidt et al. as shown in figure 3.3. Both evolutions are actually an approximation and today very different models would be used (see paragraph 3.1.2), but since no luminosity function using a more realistic evolution is available, the evolution model $(1 + z)^{6}$ will be used in further calculations since it is still more realistic than the $e^{5\tau}$ function (here, $\tau$ is the lookback time). It is certainly a fact that a function with a more realistic evolution function will have to be applied in future work (see section 3.5). The approximation made in [Win84] is based on the function in [Sch72] with the evolution $E(z) = (1 + z)^{6}$. It is shown as a dot-dashed line in figure 3.3. The radio luminosity is the integral of product of the measured luminosity per frequency $P_{\text{rad}}$ multiplied and the power law mentioned above:

$$ L_{\text{radio}} = \int_{0}^{3} \text{GHz} \, d\nu P_{\text{rad}} \left( \frac{\nu}{\text{GHz}} \right)^{-0.5} = 2 \cdot P_{\text{rad}} \left( \frac{\nu}{\text{GHz}} \right)^{0.5} \bigg|_{\nu=0.5\text{GHz}} \cdot (3.8) $$

The given data points have to be fitted with an appropriate function. The fit has been made with an ansatz

$$ n = \frac{\alpha}{(1 + \beta P_{\text{rad}})^{\gamma}} $$

with the result shown in figure 3.2.

The fit parameters are determined to be

$$ \alpha = 7.00 \left[ \frac{1}{\text{Gpc}^3} \right] $$

(3.10)
Figure 3.2: The integral luminosity function at 0.5 GHz [Sch72] (using the evolution $(1 + z)^6$) is fitted by the function $\alpha \frac{x}{(1 + \beta P_{\text{rad}})^{\gamma}}$. The parameters of the fit are $\alpha = 7.00$, $\beta = 3.02 \cdot 10^{-27}$ and $\gamma = 2.02$.

\begin{align*}
\beta &= 3.02 \cdot 10^{-27} \text{ [Hz/W]} \\
\gamma &= 2.02.
\end{align*}

The integral luminosity function is consequently described by the power law

\begin{equation}
n = \frac{7.00}{(1 + 3.02 \cdot 10^{-27} P_{\text{rad}})^{2.02}} \left[ \frac{1}{Gpc^3} \right].
\end{equation}

**Using the jet-disk symbiosis** Applying the neutrino production model near the AGN core (where protons from the inner jet interact with photons from the disk to finally produce neutrinos, see section 1.2.7), the disk luminosity (which corresponds to the luminosity of the blue bump) has to be calculated from the measured radio luminosity. To convert the radio luminosity into a disk luminosity, the jet-disk symbiosis model by Falcke et al. [FMB95] is used. For compact cores, the details are explained in section 1.2.8. Here, the model is used for extended, optically thin sources. According to Falcke et al. [FB95], the jet and the disk of an AGN are a symbiotic system. The relation between the radio luminosity of the lobes and the
Figure 3.3: Radio luminosity function according to Schmidt (1972, [Sch72]) depending on the power $P_{1.4}$ at 1.4 GHz. Dashed line: $evolution = 10^{5.7}$, solid line: $evolution = (1 + z)^6$, dash-dotted line: approximation by Windhorst [Win84].

disk luminosity is\(^2\)

$$P_{\text{lobe}} = 1.8 \times 10^{34} \frac{\text{erg}}{s \cdot \text{Hz}} \left(\frac{\text{GHz}}{\nu}\right)^{0.5} \beta_j^{1/4} \cdot P_{-12}^{1/4} \cdot x_{e, 100} \cdot \left(\frac{q_j}{L_{46}}\right)^{3/2} \cdot \gamma_{j, 5}^{7/4} \cdot u_3^{3/4} \cdot \left(\frac{q_j}{L_{46}}\right)^{3/2}. \quad (3.14)$$

This result is based on the investigation of a quasar sample, described in detail in [FB95]. The relation between radio and disk luminosity leading to equation (3.14) is shown in figure 3.4.

\(^2\)still for extended sources!
The parameters which occur in this formula are given below.

**PARAMETER LIST**

- $\beta_j$: The jet velocity in units of $c$ which can be assumed to have a value of $\beta_j \approx 1$.
- $P_{-12}$: The external radiation pressure at the AGN is given by $P_{ext} = P_{-12} \cdot 10^{-12}$ erg/cm$^3$. The pressure evolves with the time due to the expansion of the Universe. The pressure parameter depends on the redshift as $P_{-12} = (1 + z)^3 \cdot P_{-12}^{0}$, where $P_{-12}^{0}$ is the pressure of the Universe at $z = 0$. $P_{-12}^{0}$ is set to $P_{-12}^{0} = 1$.
- $x_{e,100}$: Let the ratio between the relativistic electron density and the total number density of protons be $x_e$ and the minimum Lorentz factor of the relativistic electron population divided by 100 $\gamma_{e,100}$. The modified electron density is then $x_{e,100} = x_e^{p-1}$. $p$ is a parameter. $x_{e,100} \approx 1$ is reasonable according to Falcke et al. [FB95].
- $q_{j/1}$: The total jet power $Q_{jet}$ in units of the disk luminosity, with a given value of $q_{j/1} = Q_{jet}/L_{disk} = 0.15^{-0.2}$.
- $L_{46}$: The disk luminosity per $10^{46}$ erg/s: $L_{46} = L_{disk}/(10^{46}$ erg/s).
- $\gamma_j,5$: The jet’s Lorentz Boost factor, divided by 5. It is assumed to be $(6 \pm 2)/5$.
- $u_3$: Ratio between the total energy density in the jet and the magnetic energy density, divided by a factor three. It is set to $u_3 = 1$ [FB95].

Using the parameters as mentioned in the list above and considering the fact that the data scatter within a factor three (see also figure 3.4), relation (3.14) is given as

$$L_{\text{radio}} = \int_{0}^{6\,\text{GHz}} dv P_{\text{lobe}} = (1.08^{+2.15}_{-0.72}) \cdot 10^{32} \cdot (1 + z)^{3/4} \cdot L_{39}^{3/2}. \quad (3.15)$$

The result can be inserted in the fit function (3.13), using a minimum spectral index of $\alpha_{\text{spect}} = 0.5$ and converting SI-units to cgs-units (1 W = 10$^7$ erg/s). The radio luminosity per frequency is therefore given as $P_{\text{rad}} = 10^{-16} \cdot L_{\text{radio}}/\sqrt{2}$. Consequently, the integral luminosity function depending on the luminosity (in units of $L_{39}$) is given by

$$n(L_{39}) = \frac{7.00}{(1 + 2.30^{+1.60}_{-1.33} \cdot 10^{-11} \cdot (1 + z)^{3/4} \cdot L_{39}^{3/2})^{2.62} \left[ \frac{1}{\text{Gpc}^2} \right]} \cdot (3.16)$$

Finally, the differential luminosity function is determined by differentiating the integral distribution:

$$\frac{dn}{dL_{39}} = 4.87^{+9.75}_{-3.25} \cdot 10^{-10} \cdot \sqrt{L_{39}} \cdot (1 + z)^{3/4} \left[ \frac{1}{\text{Gpc}^3 \text{ erg/s}} \right] \cdot \left(1 + 2.30^{+1.60}_{-1.33} \cdot 10^{-11} \cdot (1 + z)^{3/4} \cdot L_{39}^{1.5} \right)^{3.02} \quad (3.17)$$
Figure 3.4: Monochromatic total quasar luminosity. *Boxes are radio intermediate quasars (RIQ), black dots: radio weak point sources, dots with shades: radio weak sources with extended emission, grey dots with cross: radio weak quasars with $z > 0.6$, grey dots without cross: radio weak quasars from SM89, grey open circles with cross: radio loud quasars with $z > 0.5$, grey open circles without cross: radio loud quasars from SM89, circles and boxes with grey background: flat spectrum sources. Crosses are FR-II galaxies. A general conversion factor was used by Falcke et al. [FB95] to obtain the disk luminosity for FR-II galaxies. This factor is based on the analysis to the jet power [RS91].
The AGN distance distribution

The spatial AGN density $\psi(z)$ increases exponentially according to recent measurements [MHS00, MPD98]. At some distance $z \approx 1.7$, there is a break in the exponential function and the slope changes:

$$
\psi(z) = \begin{cases} 
A e^{az} \frac{1}{Gpc^3} & \text{for } z < 1.7 \\
B e^{bz} \frac{1}{Gpc^3} & \text{for } z > 1.7 
\end{cases} 
$$

(3.18)

Two different models [MHS00, MPD98] will be discussed in this calculation, both having the same features as in equation (3.18) but with slightly different parameters $A$, $a$, $B$ and $b$.

First model by Miyaji et al. [MHS00] To estimate the number of AGN per comoving volume at a certain redshift $z$, measurements of AGN up to redshifts of $z \approx 5.4$ have been analyzed by Miyaji et al. [MHS00] with the result shown in figure 3.5. In that figure, four samples are shown where the circles and the tri-

![Figure 3.5: The AGN distance distribution [MHS00]. Circles and triangles are ROSAT X-Ray luminosity data analyzed by Miyaji et al. squares are optical magnitude data ($M_B < -26.0$) analyzed by Schmidt et al (1995) while the star shaped points are radio luminosity data by Shaver et al. (1999) angles represent the ROSAT data analyzed by Miyaji et al.. With the ROSAT satellite experiment, AGN were observed at X-ray frequencies with luminosities...](image-url)
\[ L_X > 10^{44.5} \text{ erg/s}, \] the triangles are analyzed assuming an Einstein-de-Sitter Universe \((\Omega_m = 1.0, \Omega_\Lambda = 0.0)\) and the circles represent a \((\Omega_m = 0.3, \Omega_\Lambda = 0.0)\) Universe which is much more realistic according to recent data release by various CMB experiments (e.g. WMAP [S^+_03a]) and the Supernova Cosmology Project [PS97]^3. The other two data sets are for comparison with analyses in the optical (squares) and in the radio (stars). The ROSAT data can be fitted by an exponential function with a change in the exponent at \(z = 1.7\). The fits are shown in figure 3.6. The fits for the two different analyses are

\[
\psi(z, \Omega_m = 1.0) = \begin{cases} 
(11.66 \pm 1.92) e^{(2.81 \pm 0.18) z} \left[ \frac{h_{50}}{\text{Gpc}} \right] & \text{for } z < 1.7 \\
(1.27 \pm 0.45) 10^3 e^{(0.05 \pm 0.26) z} \left[ \frac{h_{50}}{\text{Gpc}} \right] & \text{for } z > 1.7 
\end{cases}
\]

\[
\psi(z, \Omega_m = 0.3) = \begin{cases} 
(11.63 \pm 1.72) e^{(3.00 \pm 0.14) z} \left[ \frac{h_{50}}{\text{Gpc}} \right] & \text{for } z < 1.7 \\
(1.64 \pm 0.32) 10^3 e^{(0.09 \pm 0.21) z} \left[ \frac{h_{50}}{\text{Gpc}} \right] & \text{for } z > 1.7 
\end{cases}
\]

In this context, \(h_{50}\) is the Hubble parameter in units of 50 km/(s·Mpc). As expected, the deviation from the fit parameters is quite large for higher redshifts \((z > 1.7)\). In the following calculations, the \((\Omega_m = 0.3, \Omega_\Lambda = 0.0)\) distribution is used, normalized to \(\Psi(z = 0) = 1\): Since the luminosity function is already normalized to \(z = 0\), multiplying it with the distance distribution gives the AGN density depending on luminosity and redshift.

\[
\psi(z, \Omega_m = 0.3) = \begin{cases} 
(1.00 \pm 0.15) e^{(3.00 \pm 0.14) z} & \text{for } z < 1.7 \\
(164 \pm 63) e^{(0.09 \pm 0.21) z} & \text{for } z > 1.7 
\end{cases}
\]

**Second model by Madau et al. [MPD98]** A second AGN distribution model is discussed by Madau et al. in [MPD98]. This model actually deals with the star formation rate of AGN but it can, according to Wang et al. [WYT03], be considered equivalent to the AGN distance distribution. The result is shown in figure 3.7. The data points are taken from Steidel et al. [Ste99]. The data are best fitted (in the model of equation (3.18)) by \(b = -1\) and this value will be taken for further calculations. Calculations concerning GRBs in [Pug00] confirm the predictions by [MPD98]. Table 3.1 shows the different values for the parameters of the distance distribution used by Miyaji respectively Madau.

<table>
<thead>
<tr>
<th>parameter</th>
<th>Model 1, with (\Omega_m = 0.3, \Omega_\Lambda = 0)</th>
<th>Model 2, with (\Omega_m = 1.0, \Omega_\Lambda = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>((1.00 \pm 0.15) / h_{50}^2)</td>
<td>1.00</td>
</tr>
<tr>
<td>B</td>
<td>((164 \pm 63) / h_{50}^2)</td>
<td>138</td>
</tr>
<tr>
<td>(a)</td>
<td>(3.0 \pm 0.14)</td>
<td>2.9</td>
</tr>
<tr>
<td>(b)</td>
<td>(0.09 \pm 0.21)</td>
<td>-1</td>
</tr>
</tbody>
</table>

Table 3.1: Comparison of the parameters in the AGN distribution models of Madau [MPD98] and Miyaji [MHS00].

\(^3\)see also section 1.3.
Figure 3.6: The AGN distribution by [MHS00] is fitted by an exponential function with a bending point at \(z = 1.7\). Fit parameters are given in equations 3.19 and 3.20.

### 3.1.3 Cosmological tools

This section will give a brief overview of the cosmological tools used in the calculations. A detailed description of the cosmology is given in chapter 1.3 where the motivation for the cosmological parameters \(\Omega_k = 0, \Omega_m = 0.27\) and \(\Omega_\Lambda = 0.73\) is summarized.

The comoving volume per redshift, \(dV_c/dz\) in a \(\Lambda \neq 0\) and \(\Omega_k = 0\) cosmology is given by [CPL92]

\[
\frac{dV_c}{dz} = \frac{4\pi}{(1 + z)^2} \left( \frac{c}{H_0} \right)^3 d_L^2 \left[ (1 + z)^2 \cdot (1 + \Omega_m \cdot z) - \Omega_\Lambda \cdot z \cdot (2 + z) \right]^{-1/2} .
\] (3.22)

The luminosity distance, \(d_L\), can be expressed in terms of redshift as follows [CPL92]:

\[
d_L = \left( \frac{1 + z}{c} \right) \cdot \frac{c}{H_0} \cdot I(z)
\] (3.23)

with

\[
I(z) := \int_0^z \left[ (1 + z')^2 \cdot (1 + \Omega_m z') - \Omega_\Lambda z' (2 + z') \right]^{-1/2} dz' .
\]

Thus, the product of \(dV_c/dz \cdot 1/(4\pi d_L^2)\) is given as

\[
\frac{dV_c}{dz} \frac{1}{4\pi d_L^2} = \frac{1}{(1 + z)^2} \cdot \left( \frac{c}{H_0} \right) \left[ (1 + z)^2 \cdot (1 + \Omega_m \cdot z) - \Omega_\Lambda \cdot z \cdot (2 + z) \right]^{-1/2} .
\] (3.24)
3.1. Ingredients

Figure 3.7: Star formation rate according to Madau. Data taken from Steidel. For the current calculation $a_2 = 1$ was used [Pug00].

It has to be noted that the factor $1/(4\pi d_L^2)$ is given in units of $1/(sr \cdot cm^2)$ while $dV_c/dz$ is given in units of $Gpc^3$. Using that $c/H_0 \approx 3Gpc/h$ (with $H_0 = 100 \cdot h \cdot km/(s \cdot Mpc)$ and $c \approx 3 \cdot 10^8$ km/s) and converting parsec into centimeters in the luminosity distance, $1 \text{pc} = 3.0856 \cdot 10^{18}$ cm, the result is

$$
\frac{dV_c}{dz} = \frac{3.15 \cdot 10^{-55}}{(1+z)^2} \cdot [(1+z)^2 \cdot (1 + \Omega_m \cdot z) - \Omega_A \cdot z \cdot (2+z)]^{-1/2} \left[ \frac{Gpc^3}{cm^2 \cdot sr} \right] .
$$

Furthermore the cosmological expansion of the neutrino energy due to the expansion of the Universe has to be considered:

$$
E_\nu = E_\nu^0 \cdot (1 + z) .
$$

$E_\nu^0$ is the energy of the particle when it arrives at the detector.

3.1.4 Integration limits

The differential flux is integrated over luminosity and redshift. The limits for the $z$-integration are taken to be

$$
z_{\text{min}} = 0 \quad \quad \quad z_{\text{max}} = 6 .
$$

The minimum is an trivial choice while the maximum is taken at $z = 6$ based on the assumption that the structure formation at that redshift is at a stage where the first Active Galactic Nuclei in the Universe are produced. Furthermore, the spatial distributions used are based on samples with a highest redshift of around $z \approx 6$. 
The maximum disk luminosity limit is also implicitly given by the highest observed luminosity at \( L_{39}^{\max} = 10^8 \). The lower luminosity limit depends on two facts:

1. The jet-disk symbiosis model is valid for FR-II galaxies which have luminosities of \( L_{39} > 10^7 \). It will be assumed that only FR-II galaxies produce high-energy neutrinos. This is reasonable since FR-II galaxies are much more luminous in the radio than FR-I galaxies. The radio power, however, is assumed to be strongly correlated with particle acceleration.

2. The maximum proton energy which can be produced by an AGN is connected with the luminosity according to Lovelace et al. [Lov76]:

\[
E_p^{\max} = C \cdot \sqrt{L_{39}}. \tag{3.26}
\]

Taking the highest observed energy so far, \( E_p \approx 10^{21} \) eV and the maximum luminosity, \( L_{39}^{\max} = 10^8 \), the constant \( C \) is determined to be \( C = 10^6 \) GeV. Thus, the lower luminosity limit of an AGN contributing to the flux at a certain proton energy is

\[
L_{39}^{\min} = \frac{E_p^2}{C^2}. \tag{3.27}
\]

as long as the relation exceeds the absolute lower luminosity for FR-II galaxies, \( L_{39}^{\text{absmin}} = 10^7 \).

The global luminosity limit is therefore

\[
L_{39}^{\min} = \begin{cases} 
10^7 & \text{for } \frac{E_p^2}{C^2} < 10^7 \\
E_p^2/C^2 & \text{for } \frac{E_p^2}{C^2} > 10^7 
\end{cases} \tag{3.28}
\]

and the energy cutoff lies at \( E_p^2/C^2 = 10^7 \), that is \( E_p = 10^{11.5}\text{GeV} \). Assuming neutrino production through photo-meson decays, the ratio between \( E_p \) and \( E_\nu \) is taken to be 20/1 [WB99] so that the neutrino energy cutoff is approximately

\[
E_\nu^{\text{cut}} \approx 10^{10} \text{ GeV} \tag{3.29}
\]

in the comoving frame.

### 3.2 Calculation

The integral, isotropic neutrino flux is determined by multiplying the AGN density (depending on \( z \) and \( L \)) with the comoving volume and the generic AGN spectrum. The decrease of the flux with cosmological distance is taken into account using a factor \( 1/(4\pi d_L^2) \). Finally, the differential flux is integrated over redshift and luminosity:

\[
\Phi(E_\nu) = \int_0^6 \int_{L_{39}^{\min}(z)}^{L_{39}^{\max}} dz \, dL \left( \frac{dN}{dE_\nu} \cdot \frac{dn}{dL} \cdot \Psi \cdot \frac{dV_c}{dz} \cdot \frac{1}{4\pi d_L^2} \right). \tag{3.30}
\]

The luminosity integral has to be evaluated first because the lower luminosity limit is redshift dependent. The integrand can be written as a product of a function which
is only dependent on the redshift, \( F(z) \), and a function that depends on both the luminosity and the redshift, \( G(L_{39}, z) \):

\[
\Phi(E_{\nu}) = \int_{0}^{6} \int_{L_{39}^{\text{min}}}^{L_{39}^{\text{max}}} dz dL_{39} \cdot F(z) \cdot G(L_{39}, z). \tag{3.31}
\]

The two functions are given as

\[
F(z) = (1.38 \pm 1.33) \cdot 10^{-15} E_{\nu}^{-2}(z) \Psi(z)(1 + z)^{-2} \cdot \left[ (1 + z)^2 \cdot (1 + \Omega_m z) - \Omega_A \cdot z \cdot (2 + z) \right]^{-1/2} \tag{3.32}
\]

\[
G(L_{39}, z) = 4.87_{-3.35}^{+9.75} \cdot 10^{-10} \cdot \frac{L_{39}^{3/4} (1 + z)^{3/4}}{\left( 1 + 2.30_{-0.23}^{+0.60} 10^{-11} (1 + z)^{3/4} L_{39}^{1.5} \right)^{3.62}} \tag{3.33}
\]

The integration is done numerically by first integrating \( G(L_{39}, z) \) with the limits from section 3.1.4 and then the product between the resulting function and \( F(z) \) can be integrated. The result is discussed in section 3.4.
3.3 Constraints and limits on the flux

The most constraining limit on the extraterrestrial neutrino flux is given by the analysis of the AMANDA data from 2000 [Gee03]. Figure 3.8 shows the measured atmospheric neutrino spectrum. The solid lines represent the predicted atmospheric horizontal (upper line) and vertical (lower line) flux. The spectrum is measured up to $E_\nu \approx 100$ TeV. The sensitivity which can be reached by the AMANDA-II detector (’00-’03) is at approximately $\Phi \cdot E_\nu \approx 10^{-7}$ GeV/(sr $\cdot$ s $\cdot$ cm$^2$) [Hal03].

![Figure 3.8: Measurement of the atmospheric neutrino flux up to $\sim 10^5$ GeV [Gee03]. The data fit the predicted atmospheric neutrino flux - the upper curve gives the horizontal flux while the lower is the vertical flux [VZ80]. Triangles upward are Frejus data [D1995] for comparison while triangles downward are the recent AMANDA-II data.](image_url)
Figure 3.9: The $z$ dependence in the integral for $E < E_{\text{cut}}$ with a maximum at $z=1.7$. The calculation is done using using standard parameters. The solid line is the analysis including the distance distribution by Miyaji et al., the dashed line represents the calculation using the model by Madau et al.. In the second case, the differential flux decreases more rapidly since the exponential function has a negative exponent at $z > 1.7$ instead of a positive exponent as in the first model.

3.4 Results and Conclusions

The (differential) $z$-dependence of the flux is given in figure 3.9. Here, the flux per redshift is shown for standard parameters as given in section 3.1. The maximum is around $z = 1.7$ where the spatial AGN distribution has its maximum. Thus, the dominant contribution to the quasar neutrino flux comes from relatively nearby AGN.

The flux per redshift decreases more rapidly using Madau’s model, since the distance distribution is assumed to decrease exponentially beyond the critical redshift at $z = 1.7$. On the other hand, Miyaji’s model assumes a slightly increasing exponential function at that point (see section 3.1).
Figure 3.10: The neutrino flux at $E_\nu < E_\nu^{\text{cut}}$, depending on the upper redshift integration limit. Solid line: Calculation using the Miyagi model, dashed line: Using Madau’s model. The integration is done up to $z = 6$, since the data of both AGN distributions reach up to $z=6$. The contribution above is very small at $z > 6$ which makes this part negligible as can be seen in figure 3.9.

The redshift integration is done up to $z_{\text{max}} = 6$ because of two reasons: First of all, the statistics in the models of the distance distribution reaches up to redshifts of $z \approx 5 - 6$ so that it is reasonable to integrate up to these redshifts. Secondly, the flux decreases rapidly with redshift so that the contribution is less than 1% beyond $z = 6$. The flux with a varying upper integration limit is shown in figure 3.10 (using standard parameters).

The quasar neutrino spectrum is calculated for the standard parameters and for a lower and upper limit (using the errors from the input functions given in section 3.1). The neutrino flux for the different distribution function models are shown in figure 3.11 and 3.12. Due to neutrino oscillations, each neutrino flavor contributes equally with 1/3. The result of the calculation done in section 3.2 is therefore divided by a factor of three in the plots, since the AMANDA data only include muon neutrinos. The strong deviations from the standard prediction are possibly due to errors in the
3.4. Results and Conclusions

prediction of the generic energy spectrum and in the jet-disk symbiosis model. The characteristics are the same for both distance distribution models: Up to the cutoff energy $E_{\nu}^{\text{cut}} \approx 10^{10}$ GeV the flux decreases with $(E_{\nu}^0)^2$:

$$
\Phi(E_{\nu}^0 < E_{\nu}^{\text{cut}}) \cdot (E_{\nu}^0)^2 = \begin{cases} 
2.049 \cdot 10^{-7} \text{ GeV}/(sr \cdot s \cdot cm^2) & \text{Miyaji} \\
1.190 \cdot 10^{-7} \text{ GeV}/(sr \cdot s \cdot cm^2) & \text{Madau} .
\end{cases}
$$

(3.35)

At $E_{\nu} = E_{\nu}^{\text{cut}}$, the cut condition $L_{\text{min}} = E_{\nu}^2/C^2$ begins to exceed the absolute lower luminosity limit, $E_{\nu}^2/C^2 \geq 10^7$. For even higher energies, the cut condition will also exceed the upper luminosity, $E_{\nu}^2/C^2 \geq 10^8$. This happens at $E_{\nu}^{\text{cut abs}} \approx 10^{10.7}$ GeV.

The calculations based on the two different AGN distribution models differ in normalization. Regarding the error in the calculation, the small differences due to the use of the two models are barely significant. Only for the Miyaji model an explicit error can be given.

Looking at figure 3.11 and 3.12, the limit given by the measurement of the atmospheric neutrino flux is not violated with standard parameter choice. The calculation in figure 3.11 is done using Miyaji’s distance distribution model while figure 3.12 is the calculation in Madau’s model. Measuring the neutrino spectrum up to slightly higher energies would answer the question if the standard parameter choice can be excluded. The prediction seems very promising, because for the standard parameter set a significant extraterrestrial neutrino signal can already be expected at energies of approximately $10^6 - 10^7$ GeV.

The lower limit of the calculation gives an estimation of the lower limit of the neutrino flux from quasars as it can be seen in figure 3.13. The minimal quasar neutrino flux is about two orders of magnitude lower than the AMANDA limit:

$$
\Phi(E_{\nu}^0 < E_{\nu}^{\text{cut}}) \cdot (E_{\nu}^0)^2 = \begin{cases} 
1.002 \cdot 10^{-9} \text{ GeV}/(sr \cdot s \cdot cm^2) & \text{Miyaji’s model} \\
0.9701 \cdot 10^{-9} \text{ GeV}/(sr \cdot s \cdot cm^2) & \text{Madau’s model} .
\end{cases}
$$

(3.36)

Since this is the worst case scenario, there is a good chance for neutrino observation from radio quasars for the next generation neutrino telescopes. In comparison, the expected sensitivity of ICRCUBE is plotted in figure 3.13. In the worst case scenario, ICRCUBE would not able to detect a signal from radio quasars with the sensitivity given above, since the lower neutrino flux bound is about half an order of magnitude lower than the sensitivity. The sensitivity above is, however, calculated for approximately one year of operation. Thus, even in the worst case scenario, the quasar neutrino flux could be seen if the operation time would be longer than a year. ICRCUBE is sensitive to both electron and muon neutrinos which increases the probability for the detection of the calculated flux. Furthermore, looking at potential FR-II galaxies directly increases the probability of detecting an extraterrestrial neutrino signal from radio quasars. The spectrum here has been calculated isotropically and any directional information has not been considered. The question of the neutrino spectrum from extraterrestrial sources can thus presumably be answered by the upcoming telescopes and also whether or not radio quasars contribute significantly.
Figure 3.11: The AGN neutrino flux in the Miyaji model. The flux is multiplied by $(E^0_\nu)^2$. The three predictions are upper limit, standard parameter set and lower limit (counted from above). Note that this model includes all produced neutrinos. The data of the atmospheric neutrino flux only includes muon and antimuon-neutrinos (in principle, AMANDA can also detect electron- and muon-neutrinos). The muon (and antimuon-) neutrinos contribute with approximately 1/3 concerning oscillations. The AMANDA sensitivity is indicated as $\Phi \cdot E^0_\nu \approx 10^{-7}$ GeV/(sr \cdot s \cdot cm^2).
Figure 3.12: The AGN neutrino flux in the Madau model. The flux is multiplied by $E_\nu^2$. The AMANDA-II data are again indicated. Lines are explained in figure 3.11.
Figure 3.13: The lower limit of the quasar flux is approximately equal in both AGN distribution models. The ICECUBE sensitivity is expected to be about $\Phi \cdot E_\mu^2 \sim 10^{-9}$ GeV/(sr s cm$^2$) [Hal03].
Looking at the normalization of the radio quasar neutrino flux, it can be seen that the prediction for the FR-II galaxy flux is so high that FR-I galaxies can hardly contribute significantly to the neutrino flux. Otherwise, the experimental constraints would be violated. That implies an almost vanishing neutrino flux contribution by FR-I galaxies for the standard parameter set although this type of galaxies appears more often in the Universe⁴.

3.5 Outlook

„Ich habe auf eine geringe Vermuthung eine gefährliche Reise gewagt und erblicke schon die Vorbebürge neuer Länder. Diejenigen, welche die Herzhaftigkeit haben die Untersuchung fortzusetzen, werden sie betreten und das Vergnügen haben, selbige mit ihrem Namen zu bezeichnen.”

Immanuel Kant, aus: „Allgemeine Naturgeschichte und Theorie des Himmels"

"On the basis of a slight assumption I have undertaken a dangerous journey, and I already see the promontories of new lands. Those people who have the resolution to set forth on this undertaking will enter these lands and have the pleasure of designating them with their very own names.”

Translation: Ian C. Johnston

"Utifrån ett våg antagande har jag vågat en farlig resa och ser nu redan nya länders bergstoppar. De som har modet att fortsätta undersökningen kommer att bestiga dessa och ha nöjet att beteckna dem med sitt eget namn.”

Översättning: Brigitte Mral

There are many ideas on how to improve and extend the quasar model. The following restrictions were used during this calculation:

- The model is based on the investigation of extended (that implies optically thin) sources.

- A simple $E^{-2}_{\nu}$ spectrum was assumed. The spectral index can differ slightly from the value used since no experimental evidence for a pure $E^{-2}$ spectrum can be given yet. Furthermore, the power law is not necessarily valid at very low energies.

- $p - \gamma$ reactions in the vicinity of the AGN core are believed to produce mesons which finally decay into neutrinos. $p - p$ interactions are not investigated which are possible in the footring of the AGN jets.

- The flux was assumed to be isotropic while it is really a flux from point sources.

The model can be extended by investigating the restrictions made above. My list of Things still to be done is based on these

⁴This can be seen in the luminosity functions presented by [Win84] in figure 3.1.
1. The luminosity function from section 3.1 uses an approximative evolution function which is actually only valid for small redshifts \((z < 1)\). The next step in the investigation of the quasar neutrino flux is therefore to use a luminosity function with a more realistic evolution function.

2. Quasars make up only approximately 10\% of all radio galaxies. It would be interesting to analyze a sample of radio galaxies and see how large the neutrino flux is on the whole.

3. The model for the quasar neutrino flux which is discussed here is constrained to Active Galactic Nuclei with strongly extended emission. A model for compact cores can be developed if the corresponding distributions and RLFs are given.

4. Since some of the models used are from the early 1990s or even before, the cosmology used does not always fit today’s measurements of the cosmological parameters. Although a lambda cosmology should not change the spectrum significantly\(^5\), it is preferable to match the cosmological parameters in the models used to modern cosmology.

5. Finally, it would in principle be possible to investigate a sample of FR-I galaxies. Following the basic argumentation given above, the contribution from FR-I galaxies should be some orders smaller than the flux from FR-II galaxies. The model can thus be embedded into an entire evolution model of AGN.

For my diploma thesis, I will try to give an answer to many of these questions. Subsequent to the calculations of the radio galaxy neutrino flux, I will investigate the resulting neutrino rates which can actually be observed in different (future and current) neutrino experiments. Here, the investigation is not restricted to the flux calculated within the scope of my diploma thesis, but different neutrino models (concerning AGN, GRB and more) can also be considered.

\(^5\)A slight change in normalization should be seen.
Appendix A

Overview of the calculation results

A summary of all the functions used for the calculation of the isotropic AGN neutrino flux is given in this section. Different functions have been calculated with various values for the cosmological parameters $h$ (Hubble parameter), $\Omega_m$ and $\Omega_\Lambda$. Today, these values are believed to be $h \approx 0.71$ and $\Omega_0 \approx 0.15$ (see also section 1.3).

- The generic energy spectrum

$$\frac{dN}{dE} = (4.369 \pm 4.225) \cdot 10^{39} \cdot L_{39} \cdot E_{\nu}^{-2} \left[ \frac{1}{\text{GeV} \cdot s} \right].$$

No special choice of cosmological parameters is assumed here.

- The luminosity function by Schmidt et al. [Sch72] is given as

$$n = \frac{700}{(1 + 3.02 \cdot 10^{-27} P_{\text{rad}})^{2.02}} \left[ \frac{1}{\text{Gpc}^3} \right].$$

The used Hubble parameter is $h = 0.5$ and the matter and vacuum energy density are taken to be $\{\Omega_m,\Omega_\Lambda\} = \{1.0, 0.0\}$.

- The relation between radio and disk luminosity by Falcke et al. [FMB95] for strongly extended sources is

$$L_{\text{radio}} = \int_0^{5\,\text{GHz}} d\nu P_{\text{obs}} = (1.08^{+2.15}_{-0.72}) \cdot 10^{32} \cdot (1 + z)^{3/4} \cdot L_{39}^{3/2}.$$ 

The cosmological parameters which are used in [FMB95] are $\{h, \Omega_m, \Omega_\Lambda\} = \{0.5, 1.0, 0.0\}$. Since two different luminosities are correlated here, the choice of parameters should not change the result very much. The error in the normalization is due to a scattering factor of three in the data.

- Inserting the luminosity relation in the model for the luminosity function by [Sch72], the result for the differential luminosity function is

$$\frac{dn}{dL_{39}} = 4.87^{+9.75}_{-3.25} \cdot 10^{-10} \cdot \sqrt{L_{39}} \cdot (1 + z)^{3/4} \left[ \frac{1}{\text{Gpc}^3 \text{ erg/s}} \right].$$
• The distance distribution is given in two models which are discussed in section 3.1.2. The structure of the function is the same for both models:

\[
\psi(z) = \begin{cases} 
A e^{a z} & \text{for } z < 1.7 \smallskip 
B e^{b z} & \text{for } z > 1.7
\end{cases}
\]

The first model by Miyaji at al. [MHS00] is given for the following set of cosmological parameters: \( \{h, \Omega_m, \Omega_\Lambda\} = \{0.71, 0.3, 0.0\} \) The model by Madau et al. [MPD98] uses the cosmology as \( \{h, \Omega_m, \Omega_\Lambda\} = \{0.71, 1.0, 0.0\} \).

• For the luminosity distance and the comoving volume, a \( \Omega_k = 0 \) cosmology with \( h = 0.71 \) and \( \{\Omega_m, \Omega_\Lambda\} = \{0.27, 0.73\} \) is used:

\[
\frac{dV_c}{dz} = 4\pi d_L^2 \frac{3.15 \cdot 10^{-55}}{(1+z)^2} \left[(1+z)^2 \cdot (1 + \Omega_m \cdot z) - \Omega_\Lambda \cdot z \cdot (2 + z) \right]^{-1/2} \left[ \frac{Gpc^3}{cm^2 \cdot sr} \right].
\]
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\end{align*} \]
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