Calculation of the AGN Neutrino Flux and of Event Rates for Large Volume Neutrino Telescopes

Diplomarbeit
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Contents

Introduction .......................................................... 1

Astroparticle Physics & Cosmology ................................. 5
  1.1 A glimpse into astroparticle physics .......................... 5
      1.1.1 The Cosmic Ray spectrum .................................. 6
      1.1.2 Acceleration processes ..................................... 7
  1.2 Atmospheric and extraterrestrial neutrino spectra ............ 10
      1.2.1 Neutrino production ....................................... 10
      1.2.2 Atmospheric neutrinos ..................................... 11
      1.2.3 Extraterrestrial neutrinos .................................. 12
  1.3 Active Galactic Nuclei (AGN) ................................ 14
      1.3.1 Basic features .............................................. 14
      1.3.2 Classification criteria ..................................... 15
      1.3.3 AGN classes ................................................ 15
      1.3.4 Morphology ................................................ 17
      1.3.5 Unification ................................................ 18
      1.3.6 Radiation processes ....................................... 18
      1.3.7 The proton spectrum ...................................... 21
      1.3.8 Jet-disk symbiosis ....................................... 22
      1.3.9 Neutrino production in AGN ............................... 24
      1.3.10 AGN as extragalactic neutrino source candidate ...... 27
      1.3.11 AGN neutrino spectra predictions ........................ 27
  1.4 Cosmology ........................................................ 30
      1.4.1 Cosmological parameters .................................. 30
      1.4.2 Cosmological distances .................................... 33
      1.4.3 The Comoving Volume ...................................... 34
1.4.4 Cosmic Microwave Background ................. 34
1.4.5 Measuring cosmological parameters .......... 35

A model for the integral neutrino flux from AGN 37
2.1 The samples and their indices .................. 37
2.1.1 The samples .................................. 37
2.1.2 The spectral and particle index of the sources 38
2.2 Ingredients of the flux calculation .............. 40
2.2.1 The generic neutrino flux .................... 41
2.2.2 AGN RLF for steep spectrum sources .......... 47
2.2.3 AGN RLF for flat spectrum sources .......... 52
2.2.4 Cosmological tools ........................... 54
2.2.5 Integration limits ............................ 54
2.3 Constraints and limits on the extragalactic neutrino flux 56
2.4 Results from the neutrino flux calculation .... 57
2.4.1 Dependence on the parameters ............... 57
2.4.2 The final spectra ............................ 63

Uncertainties in neutrino flux calculations 67
3.1 Redshift dependence ............................ 68
3.2 Neutrino flux using Willott’s RLF ............... 70
3.3 Conclusions ................................... 70

Large Volume Neutrino Experiments 73
4.1 Optical Cherenkov Light Detection .............. 74
4.2 Radio Cherenkov Radiation ...................... 76

Expected event rates for neutrino telescopes 79
5.1 Weak interactions ............................... 79
5.1.1 Neutrino nucleon scattering cross sections .... 80
5.1.2 The differential cross section ................ 81
5.1.3 Integration limits for the cross section ....... 82
5.1.4 Total cross section .......................... 83
5.2 Production of neutrino induced muons .......... 85
5.2.1 Shadow factor .............................. 85
5.2.2 Probability of producing a muon....................... 90
5.2.3 Effective Areas........................................ 91
5.3 Cascades...................................................... 93
5.3.1 Probability of detecting a cascade...................... 94
5.4 Results from the event rate calculation.................. 95
5.4.1 Detection of muon neutrinos............................ 95
5.4.2 Detection of electron neutrinos......................... 100

Conclusions..................................................... 105

Appendix......................................................... 110

A Tables......................................................... 110

B Overview of the calculation results........................ 115
  B.1 Neutrino flux from Quasars............................. 115
     B.1.1 Steep spectrum sources.............................. 115
     B.1.2 Flat spectrum sources............................... 116
     B.1.3 Cosmology.............................................. 117
  B.2 Detector Rate............................................. 118
     B.2.1 Cross sections........................................ 118
     B.2.2 Calculation............................................ 120
     B.2.3 Figures of event rates for various detector arrays 120
  B.3 Physical Constants...................................... 123
List of Figures

1.1 The all particle Cosmic Ray spectrum .......................... 6
1.2 The Hillas plot ................................................. 8
1.3 First order Fermi acceleration .................................. 9
1.4 Second order Fermi acceleration ................................ 9
1.5 Cosmic neutrino spectra [Ron00] ............................... 10
1.6 Atmospheric neutrino flux prediction .......................... 13
1.7 NGC-4261 ......................................................... 14
1.8 AGN scheme ...................................................... 19
1.9 Synchrotron radiation ........................................... 20
1.10 Scheme of the continuum of a typical AGN spectrum [CO96] 20
1.11 Particle production in AGN .................................... 24
1.12 Tree scheme for the selection of AGN neutrino source candidates 26
1.13 Muon neutrino flux predictions from AGN ........................ 28
1.14 The cosmological diagram ...................................... 32
1.15 Distance measures ............................................... 34
2.1 Histogram of the spectral indices of the steep spectrum sources .... 39
2.2 The histogram of the flat spectrum spectral indexes at 5 GHz .......... 39
2.3 Generic AGN neutrino spectrum from steep spectrum sources .......... 46
2.4 Generic AGN neutrino spectrum from flat spectrum sources .......... 46
2.5 Evolution function of the low and high luminosity populations .......... 48
2.6 RLF for $\Omega_m = 1$ and $z = 0, 1, 2$ .................................... 49
2.7 RLF for $\Omega_m = 0$ and $z = 0, 1, 2$ .................................... 49
2.8 Spatial AGN distribution [MHS00] ................................... 51
2.9 Comparison of data from ROSAT with the evolution function from [W+00] 51
2.10 RLF for flat spectrum sources [DP90] .............................. 53
2.11 Limits on the extragalactic neutrino flux .......................... 56
2.12 Dependence of the steep spectrum sources on the upper luminosity limit. 58
2.13 Variation of the steep spectrum on the lower luminosity limit. 58
2.14 Differential flux $d\Phi/dz$ of the steep spectrum sources versus $z$. 59
2.15 Variation of $z_{\text{max}}$ for steep spectrum sources. 59
2.16 Variation of the maximum luminosity of the flat spectrum flux. 60
2.17 Dependence of the flat spectrum AGN flux on the lower integration limit. 61
2.18 Differential flat spectrum AGN flux $d\Phi/dz$. 61
2.19 Dependence of the flat spectrum source flux on $z_{\text{max}}$. 62
2.20 Neutrino spectrum from steep spectrum AGN. 63
2.21 Neutrino spectrum from flat spectrum AGN. 64
2.22 AGN neutrino spectrum. 65
2.23 AGN neutrino spectrum with an index of 2. 66
3.1 Approximate redshift dependence of the neutrino flux. 68
3.2 Ratio of the $z$-dependences. 69
3.3 Integration over approximate redshift dependence of the neutrino flux. 69
3.4 Ratio between the integral results of the two models EdS and $\Omega_m = 0.3$. 70
3.5 Redshift dependence of the neutrino spectrum (Willott). 71
3.6 Neutrino spectrum in two different cosmologies. 71
4.1 Topology of a muon track (left) differs from a cascade event (right) [Kow04]. 74
4.2 Schematic drawing of the AMANDA detector. [CA02]. 78
5.1 $N \eta \rightarrow l X$. 80
5.2 Total cross section for neutrino-nucleon interactions and Glashow resonance. 84
5.3 Traveling distance of a neutrino through Earth. 86
5.4 Model of Earth layers. 87
5.5 Density of the Earth depending on the radius according to [DA81]. 87
5.6 Neutrino absorption length versus zenith angle. 88
5.7 Shadow factor ($\nu_\mu$). 89
5.8 Probability of detecting a neutrino induced muon. 91
5.9 AMANDA effective area versus muon energy according to [BA03]. 92
5.10 ANTARES effective area [Mon03]. 92
5.11 IceCube effective area [Ice04]. 93
5.12 Probability of detecting a neutrino induced cascade versus neutrino energy. 94
5.13 Differential muon rate at a zenith angle of 90° versus neutrino energy. 95
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.14</td>
<td>Event rate depending on the threshold energy of the detector.</td>
<td>96</td>
</tr>
<tr>
<td>5.15</td>
<td>Atmospheric rate of neutrino induced muons versus $\theta$.</td>
<td>97</td>
</tr>
<tr>
<td>5.16</td>
<td>Event rate versus zenith angle for AMANDA detector properties.</td>
<td>97</td>
</tr>
<tr>
<td>5.17</td>
<td>AMANDA detector rate per year and steradian versus zenith angle.</td>
<td>98</td>
</tr>
<tr>
<td>5.18</td>
<td>ANTARES detector rate per year and steradian.</td>
<td>99</td>
</tr>
<tr>
<td>5.19</td>
<td>IceCube detector rate per year and steradian.</td>
<td>99</td>
</tr>
<tr>
<td>5.20</td>
<td>Differential cascade rate per (yr km$^2$ GeV), various flux models.</td>
<td>100</td>
</tr>
<tr>
<td>5.21</td>
<td>Differential cascade rate per (yr km$^2$ GeV), chapter 2.</td>
<td>101</td>
</tr>
<tr>
<td>5.22</td>
<td>Differential cascade rate per (yr km$^2$ GeV), chapter 2 with $p = 2$.</td>
<td>102</td>
</tr>
<tr>
<td>5.23</td>
<td>Differential rate per (yr km$^2$ GeV), Mannheim [Man95].</td>
<td>102</td>
</tr>
<tr>
<td>5.24</td>
<td>Event rate for electron neutrinos, dependent on the threshold energy.</td>
<td>103</td>
</tr>
</tbody>
</table>

B.1 Parton Distribution Functions (PDFs). 119
## List of Tables

1.1 Angle distribution parameters. .................................................. 12
1.2 The AGN zoo ................................................................. 17
1.3 Relation between radio and disk luminosity ........................ 23
1.4 Current cosmological parameters ......................................... 35
2.1 Parameters for the steep spectrum RLF calculation [W+00]. .... 48
2.2 Parameters for the flat spectrum RLF [Pea85]. ....................... 53
2.3 Parameters for the determination of the lower luminosity integration limit. 55
4.1 Properties of UHE neutrino experiments (I). ...................... 73
4.2 Properties of UHE neutrino experiments (II). ....................... 74
A.1 Steep spectrum sample. ....................................................... 112
A.2 171 flat spectrum sources and their spectral index at 5 GHz. ... 114
Introduction

In the past decades, much progress has been made in confirming the standard model of particle physics with the aid of astroparticle physics. Recently, neutrino oscillations could be proved by observing the sun in the neutrino light. The field of neutrino physics can be examined with the help of cosmic accelerators up to $E > \text{TeV}$. In this thesis two main aspects of high energy neutrino physics will be pursued:

A new prediction of the neutrino flux from Active Galactic Nuclei (AGN) will be made and the neutrino induced event rate will be computed for diverse large volume neutrino detection arrays. The thesis is organized as follows. In the first chapter, an overview over the current status of astroparticle physics and cosmology is given. Here, the focus lies on neutrino production in the atmosphere and in extragalactic sources, particularly in AGN. Also, cosmological variables will be discussed.

In the 2nd chapter, flat and steep spectrum AGN sources are considered to calculate the neutrino flux from AGN. To determine the generic AGN neutrino spectrum, the jet-disk symbiosis is used together with the assumption that the spectrum follows a power law up a certain energy at which it starts to decrease exponentially. The power law index is assumed to be the same for protons and neutrinos. That means it is directly connected to the spectral index of the synchrotron radiation spectrum of the sources. The spectral indices are separately computed from the indices of the flat and steep spectrum sources at 5 GHz, where the observed spectrum is presumably resulting from synchrotron radiation. It can be shown that the resulting spectrum is dominated by steep spectrum sources up to energies of $\sim 10^6$ GeV. At higher energies, the flat source spectrum increases and the total spectrum is produced by these sources.

Chapter 3 gives an overview of possible uncertainties of neutrino flux models due to the choice of cosmological parameters. It is shown that with a consistent choice of parameters, the result is the same within a factor of $2 - 3$ which lies within the errors of the used models. In chapter 4, current and future neutrino telescopes are discussed.

The integral neutrino induced event rate for various detection arrays is computed in chapter 5. Neutrino-nucleon cross sections are calculated using particle distribution functions from the Particle Distribution Function Library (PDFLib, [CER]). The neutrino induced muon rate in detection arrays such as AMANDA, ANTARES and IceCube is determined for different neutrino flux models. Both conventional and charm including atmospheric neutrino flux will be examined as well as an AGN neutrino flux model by Mannheim [Man95]. Additionally, two variations of the neutrino flux calculated in chapter 2 will be discussed. The flux will be examined using a spectral index of $p = 2$ as it is common in literature and further, the calculation with a power law index computed from the spectral indices of the sources will be considered. Furthermore, the electron neutrino induced flux of cascades through the detector is evaluated for the flux models mentioned above. Here, the Glashow resonance has to be concerned in addition to charged and neutral current cross sections. At higher energies, a substantial gain of event rate is reached by looking upwards instead of using the Earth as a filter, since the effective range of the cascades is much shorter than for muons. It can be shown that an extragalactic neutrino flux component should be observed in the future, in all likelihood with future experiments like IceCube if not already with current experiments as AMANDA.
Einleitung


Die Arbeit ist wie folgt aufgebaut:
Im 1. Kapitel wird ein Überblick über den momentanen Status der Astroteilchenphysik und der Kosmologie gegeben. Das Hauptaugenmerk liegt hier auf der Neutrinoerzeugung in der Atmosphäre und in extragalaktischen Quellen, insbesondere in AGN. Außerdem werden kosmologische Parameter diskutiert.


Kapitel 3 gibt eine Übersicht über mögliche Unsicherheiten, die auf Grund der Wahl der kosmologischen Parameter auftreten. In diesem Kapitel wird gezeigt, daß das Resultat bei konsistenter Wahl der Parameter im Rahmen der Fehler der benutzten Funktionen unabhängig von der Wahl der kosmologischen Parameter ist.

Introduktion

Under de senaste decennierna har partikelfysikens standardmodell utvecklats med hjälp av astropartikelfysiken. Nyligen kunde neutrinooscillationer bevisas genom observationer av solen i neutrinojus. Åmnesområdet neutrinofysik kan undersökas mha kosmiska acceleratoror som ger energier upp till TeV. Två huvudaspekter av neutrinofysiken ska undersökas i denna uppsats. Neutrinoöfördet från Aktiva Galaktiska Kärnor (AGN) kommer att förutsågas och det neutrinoinducerade händelsesetalet för olika neutrino- tektorer av storvolym beräknas. Denna uppsats är uppdelad enligt följande:

I 1:sta kapitlet ges en översikt av det aktuella läget inom astropartikelfysiken och kosmologin. Tyngdpunkten ligger på neutrinoproduktionen i atmosfären och i extragalaktiska källor, särskilt i AGN. Dessutom diskuteras kosmologiska parametrar.


Kapitel 3 ger en översikt av möjliga osäkerheter i beräkningarna av neutrinoöfördet pga valet av de kosmologiska parametrarna. Det kan visas att resultatet är oberoende av valet inom funktionernas felmarginal.

I kapitel 4 diskuteras aktuella och framtida neutrinoexperiment.

Chapter 1

Astroparticle Physics & Cosmology

1.1 A glimpse into astroparticle physics

In 1785, Coulomb found out that air is electrically conducting. The discovery of radioactivity by Bequerell (1896) and Curie (1898) helped to explain the phenomenon since ionizing radiation was found coming from rocks containing uranium [Kra96]. Later in 1912, Viktor Hess conducted experiments in hot air balloons and discovered that the ionizing radiation was increasing with the height of the balloon in the atmosphere. Further experiments made by Kohlhörster (1914) and Millikan (1926, [Mil26]) reaching a height of 15 km confirmed the extraterrestrial origin of the radiation. It was Millikan who introduced the term Cosmic Rays (CRs) for the radiation which is still in use. Since then, the Cosmic Ray spectrum has been examined in detail and it has been subdivided into two particle types: The primary Cosmic Rays are particles which are produced in distant sources and reach the Earth directly from outer space. Secondary Cosmic Rays are induced by interactions between primary (or even secondary) CRs and molecules in the atmosphere. In the early years, a lot of progress in particle physics was made through the examination of CRs. The discovery of the positron, of the muon (1937, [SS37]) and of several mesons (e.g. the pions and kaons) and hadrons (e.g. the Delta resonance and Ξ) was made in CR experiments before the field of particle physics was dominated by accelerator physics. The use of accelerators then became so successful that cosmic ray particle physics stagnated for a long while. Today it is known that the Cosmic Ray energy spectrum exceeds the upper energy limit for accelerator physics by several orders of magnitude. While man-made accelerators can reach energies of several TeV, cosmic accelerators produce particles with energies up to $10^9$ TeV. Today, elementary particle physics and astroparticle physics complement one another since the accurate results gained in accelerator physics at low energies are used in CR physics while knowledge about ultra high-energy (UHE) particles is achieved by observ-
1.1. A glimpse into astroparticle physics

Figure 1.1: The all particle Cosmic Ray spectrum. The differential flux is multiplied by $E^{2.75}$ to have a better presentation of the bending points [Rho02].

... cosmic accelerators. Also, the question of non-vanishing neutrino masses has been examined closely with much progress by CR physics: Neutrino oscillations have been observed by SuperKamiokande and SNO [Sup98, SNO02]. At least two neutrino flavors must have a mass $m_{\nu_i} > 0$ when oscillations between the flavors are detected.

Besides the neutrino oscillation/mass-question, the fundamental questions being asked in modern astroparticle physics are

- What is the origin of the CRs?
- What does the energy spectrum of the CRs look like?
- What are the acceleration processes?

The current status in answering these questions will be presented here very shortly.

### 1.1.1 The Cosmic Ray spectrum

Today it is known that the Cosmic Ray spectrum at energies $E > 1$ GeV consists of approximately 90% protons, 10% $\alpha$-particles, 1% heavy nuclei and 1% leptons [Rac92]. The energy spectrum is shown in figure 1.1. The particle flux is given in units $[\Phi] = 1/(\text{GeV} \cdot \text{s} \cdot \text{sr} \cdot \text{cm}^2)$. In figure 1.1, the flux is multiplied by $E^{2.75}$ so that the slopes of the different parts of the spectrum are sufficiently flat to have a good presentation. It can clearly be seen that the spectrum has two kinks. The first one at approximately $E \approx 10^{15} \text{ eV}$ is called the knee while the second one at $E \approx 10^{19} \text{ eV}$ is referred to as the ankle. All three parts can mathematically be described by a power law, $\Phi \propto E^{-\gamma}$. 
The spectral indices for the different parts of the spectrum are [Wie98]

$$\gamma \approx \begin{cases} 
2.67 & \text{for } E < 10^{15} \text{ eV} \\
3.10 & \text{for } 10^{15} \text{ eV} < E < 10^{19} \text{ eV} \\
2.75 & \text{for } 10^{19} \text{ eV} < E < 10^{21} \text{ eV}. 
\end{cases} \quad (1.1)$$

Different particles are produced with slightly different indices, so that the numbers above can be regarded as a mean value for all particles. An ansatz to explain the power law spectra is diffuse shock acceleration which will be explained in section 1.1.2. The spectral index is determined through the kinematics of the acceleration process so that different objects produce fluxes with different spectral indices. This is a possible explanation for the kinks in the spectrum since calculations for supernovae into the interstellar medium (ISM-SN) as well as pulsar winds predict acceleration up to $10^{15}$ eV while explosions of heavy star supernova into their own previous wind$^1$ can accelerate particles up to $10^{19}$ eV. A mystery in astroparticle physics is the events above $\approx 5 \cdot 10^{19}$ eV. A theory developed by Greisen, Zatsepin and Kuzmin (GZK, [Gre66]) predicts that particles with energies $E > 5 \cdot 10^{19}$ eV will interact with the Cosmic Microwave Background (CMB) and therefore have an approximative mean free path of $< 50$ Mpc. This implies that the any source producing energies at $E > 5 \cdot 10^{19}$ eV should not be too far away. However, about 20 events with energies above this limit have been detected by AGASA$^2$[Y+95] and other experiments. It has not yet been completely understood why there are such high-energy events. One idea is that Active Galactic Nuclei (AGN) of the type FR-II, which are the objects with the strongest and most effective shock waves in the Universe, may be able to produce particles which exceed the GZK limit$^3$. Energies up to $10^{20}$ eV can be reached so that the CR flux above the ankle could be explained by particle acceleration in AGN. The near FR-I galaxy M87 is also a candidate for the production of UHE particles: This might not be one of the most powerful sources, but is close to Earth so that a significant particle flux up to energies of $E \sim 10^{20}$ eV can be expected [PDR03]. Other possible sources for ultra high-energy events are Gamma Ray Bursts (GRB) and Topological Defects (TD) and Z bursts.

### 1.1.2 Acceleration processes

The first step towards understanding the Cosmic Ray spectrum is examining the possible acceleration processes. Cosmic acceleration of particles is explained by the Bottom-Up scenarios which can be divided into two groups: There are models describing compact objects accelerating particles through a one-step acceleration process. The other possibility is shock-wave acceleration where many small acceleration processes are considered with a final effective acceleration found by using statistical methods. Regardless of the acceleration process, the maximum energy that can be reached by a particle of charge $Ze$ is given by [Hil84]

$$E_{\text{max}} = \beta \cdot Z \left( \frac{B}{1 \mu G} \right) \left( \frac{R}{1 \text{ kpc}} \right). \quad (1.2)$$

Here, $B$ is the magnetic field strength of the shock front, $R$ is the size of the particle accelerating object and $\beta$ is the shock velocity in units of the speed of light. The

---

$^1$These SNS will from now on be called supernova remnants.  
$^2$Akeno Giant Air Shower Array  
$^3$For a detailed description of AGN see section 1.3.
equation is visualized in figure 1.2, the so called Hillas plot. The lines in the plot represent the energy limits for various particles. Objects below the lines are not able to accelerate the corresponding particle to energies above the GZK cutoff \( (E > 5 \cdot 10^{19} \text{ eV}) \) [Arg00].

Figure 1.2: The Hillas plot showing various objects. The logarithm of the magnetic field of the sources is shown versus the size of the objects. The lines represent proton acceleration (upper: \( \beta = 1/300 \) and middle: \( \beta = 1 \)) and iron acceleration with \( \beta = 1 \). Objects below the lines cannot accelerate the corresponding nucleon to energies above the GZK cutoff. AGN are a candidate for the so called super-Greisen events.

The first approach to stochastic acceleration was made by E. Fermi. In First order Fermi acceleration particles are accelerated in a plane shock front moving with a velocity \( V_s \) while the shocked gas moves with \( V_p \) (see figure 1.3) It describes an acceleration process in which the particle gains energy in every acceleration cycle. Second order Fermi acceleration considers a moving cloud of plasma in which the particle can also lose energy in acceleration cycles [Gai90]4. The two different acceleration processes are

---

4Second order Fermi acceleration was Fermi’s original approach to stochastic shock acceleration. First order Fermi acceleration was introduced in the 1970th.
Presented in figure 1.4 and 1.3. First order Fermi acceleration is also known as diffuse shock acceleration. Fermi acceleration leads to the observed power law spectrum. To accelerate particles to the energies found in the Cosmic Ray spectrum, strong shock waves are necessary like the ones produced in supernova remnants. The spectral index is independent of the shock kinematic. It is determined by the lifetime of the Fermi accelerator. The maximum reachable energy is determined through the strength of the magnetic field. Acceleration in supernova shocks can only result in energies up to $10^{15}$ eV which could explain the change of the spectral index at this energy. Particle acceleration in pulsarwind shocks in binary systems containing neutron stars is also possible up to $10^{15}$ eV. The main part of the Cosmic Ray spectrum beyond the knee is assumed to be produced by supernova remnants. These objects can accelerate particles up to approximately $10^{19}$ eV. There are indications of a non vanishing component of the flux beyond $10^{19}$ eV [Y+95]. However, the sources for events above $E \sim 5 \cdot 10^{19}$ eV (also called trans-Greisen events) are not yet identified. Possible sources are AGN, GRB, TD or Z bursts.

To distinguish between first order and second order Fermi acceleration, the different geometries have to be considered. Explicit calculations can be found in [Gal90] and [Pro98]. Two conclusions can be drawn from Fermi acceleration:

- The time it takes to accelerate a particle up to a certain energy increases with the energy of the initial particle.

- Limiting a certain Fermi accelerator to a lifetime $T_{\text{acc}}$, the source is characterized by a maximum energy per particle that it can produce.
1.2 Atmospheric and extraterrestrial neutrino spectra

1.2.1 Neutrino production

Neutrinos can be produced in weak decays subsequent to the production of hadrons in interactions of protons with a target. The target can be matter or photon fields [GHS95]. In particular, interactions of protons with photons and proton proton interactions are relevant for this thesis since protons are accelerated in AGN jets. These protons can interact with each other or with photons in the jet (see section 1.3):

\[
\begin{align*}
p p & \rightarrow \Delta^+ p \rightarrow p n \pi^+ \\
n p & \rightarrow \Delta^0 p \rightarrow p p \pi^- \\
p \gamma & \rightarrow \Delta^+ \rightarrow n \pi^+ \\
n \gamma & \rightarrow \Delta^0 \rightarrow p \pi^- .
\end{align*}
\]

The pions and the neutron then produce neutrinos through the decays

\[
\begin{align*}
\pi^+ & \rightarrow \mu^+ \nu_\mu \rightarrow e^+ \nu_e \overline{\nu}_\mu \\
\pi^- & \rightarrow \mu^- \overline{\nu}_\mu \rightarrow e^- \nu_e \overline{\nu}_\mu \\
n & \rightarrow p e^- \overline{\nu}_e .
\end{align*}
\]

The neutrino flux measured on Earth can be subdivided into fluxes of different origins as it is indicated in figure 1.5 [Rou00].
1.2.2 Atmospheric neutrinos

Atmospheric neutrinos are generated when protons from primary Cosmic Rays interact with nucleons from the atmosphere. The neutrino spectrum is predicted to follow a steep spectral index ($\propto E_\nu^{-3.7}$) up to energies at which charmed hadrons start contributing to the spectrum. The atmospheric neutrino spectrum has been measured up to 100 TeV by the AMANDA\textsuperscript{5} experiment. The different contributions to the atmospheric neutrino flux will shortly be discussed in this section.

The main contribution to the atmospheric neutrino spectrum at energies $E_\nu < 100$ TeV comes from decaying pions and kaons which are produced by primary protons and neutrons interacting with nucleons from the atmosphere as described in section 1.2.1. Since the decay length of pions and kaons is sufficiently large ($\tau \sim 10^{-8}$ s), a considerable fraction interacts before decaying. Thus, using a primary particle spectrum of $\propto E^{-2.7}$ for primary energies $>10^5$ GeV, the atmospheric neutrino spectrum is steeper [Rho02] by about one power. The exact description for $E_\nu < 100$ GeV is given by Honda et al. [H+95]. According to Volkova the muon neutrino spectrum at $E_\nu > 100$ GeV can be described as [VZ80]

$$
\frac{dN}{dE_\nu d\Omega} |_{\nu_\mu} (E_\nu, \theta) = \left\{ \begin{array}{ll}
0.0285 \cdot E_\nu^{-2.69} \cdot \left[ \frac{1}{1+6 E_\nu/E_\pi(\theta)} \right] & 100 \text{ GeV} \leq E_\nu < 5.4 \cdot 10^5 \text{ GeV} \\
0.48 \cdot E_\nu^{-4.04} \cdot \left[ E_\pi(\theta) + 0.89 E_{K^\pm}(\theta) \right] & E_\nu \geq 5.4 \cdot 10^5 \text{ GeV} 
\end{array} \right.
$$

and the electron neutrino spectrum is given as

$$
\frac{dN}{dE_\nu d\Omega} |_{\nu_e} (E_\nu, \theta) = \left\{ \begin{array}{ll}
0.0024 \cdot E_\nu^{-2.69} \cdot \left[ \frac{0.05}{1+1.44 E_\nu/E_{K^0}(\theta)} + \frac{11.4 E_{K^0}(\theta)}{1+1.19 E_\nu/E_\pi(\theta)} \right] & 100 \text{ GeV} \leq E_\nu < 3.7 \cdot 10^5 \text{ GeV} \\
0.0071 \cdot E_\nu^{-4.045} \cdot \left[ E_{K^\pm}(\theta) + 3.7 E_{K^0}(\theta) \right] & E_\nu \geq 3.7 \cdot 10^5 \text{ GeV} .
\end{array} \right.
$$

$E_\pi$, $E_{K^\pm}$ and $E_{K^0}$ are angle distribution parameters, depending on the zenith angle $\theta$. $\zeta$ is energy dependent as

$$
\zeta(\theta) = a(\theta) + b(\theta) \cdot \log(E_\nu) .
$$

The values of $E_\pi$, $E_{K^\pm}$, $E_{K^0}$, $a(\theta)$ and $b(\theta)$ are listed in table 1.1 [VZ80]. The ratio of the muon neutrino to antimuon neutrino flux, $R(E_\nu)$ is varying with the logarithm of the neutrino energy. It is given as [Wei93]

$$
R(E_\nu) = \frac{(dN/(dE_\nu d\Omega))_{\mu}}{(dN/(dE_\nu d\Omega))_{\mu}} = \left\{ \begin{array}{ll}
1 & E_\nu \leq 1 \text{ GeV} \\
1 + 0.3 \cdot \log(E_\nu/\text{GeV}) & 1 \text{ GeV} < E_\nu \leq 100 \text{ GeV} \\
0.6 + 0.5 \cdot \log(E_\nu/\text{GeV}) & 100 \text{ GeV} < E_\nu \leq 1000 \text{ GeV} \\
1.2 + 0.3 \cdot \log(E_\nu/\text{GeV}) & E > 1000 \text{ GeV} .
\end{array} \right.
$$

(1.3)

At $E_\nu \gtrsim 100$ TeV, atmospheric neutrinos are dominantly produced in the decay of hadrons with charm and beauty constituents (e.g. $D_0, D^+, D^-, D^+_S$ mesons and $A_C$ baryons). Due to short lifetimes ($\sim 10^{-12}$ s) these hadrons do not interact before

\textsuperscript{5}AMANDA = Antarctic Muon and Neutrino Detector Array.
1.2. Atmospheric and extraterrestrial neutrino spectra

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\cos(\theta) & 1 & 0.6 & 0.4 & 0.3 & 0.2 & 0.1 & 0.05 & 0 \\
\hline
E_\nu(\theta) & 121 & 202 & 298 & 392 & 572 & 886 & 1060 & 1190 \\
E_{K^+}(\theta) & 897 & 1500 & 2190 & 2900 & 4220 & 6540 & 7820 & 8760 \\
E_{K^0}(\theta) & 194 & 324 & 473 & 628 & 915 & 1410 & 1690 & 1890 \\
a(\theta) & -1 & -0.355 & -0.687 & -0.619 & -0.384 & -0.095 & 0.0 & 0.083 \\
b(\theta) & 0.0 & -0.23 & -0.01 & -0.007 & -0.09 & -0.165 & -0.186 & -0.215 \\
\hline
\end{array}
\]

Table 1.1: Angle distribution parameters.

decaying. This is valid up to \( E_\nu < 10^9 \) GeV. Therefore, the spectrum of the so called \textit{prompt neutrinos} follows the primary index, \( \propto E_\nu^{-2.7} \).

Figure 1.6 shows the neutrino spectrum from charged pions and kaons [VZ80, H+95]. The solid lines represent the conventional contribution from muon neutrinos, the upper line indicating the horizontal flux (\( \theta = 90^\circ \)) and the lower line indicates the vertical flux (\( \theta = 180^\circ \)). The prompt muon neutrino spectrum [MRS03] is represented by the dotted lines. Due to uncertainties in the charged neutrino spectrum because of the lacking knowledge of the behavior of the hadron cross sections at a small Bjorken variable \( x \) (see chapter 5), two lines are indicated. The dashed lines show the electron neutrino spectrum, where the upper line is again the horizontal flux and the lower line is the vertical flux.

### 1.2.3 Extraterrestrial neutrinos

The spectrum of the extraterrestrial neutrinos may have a galactic or extragalactic origin:

- **Relic neutrinos** are leftovers from approximately 1 s after the Big Bang, when neutrinos decoupled. Today, these relic neutrinos have a temperature of 1.9 K which corresponds to an energy in the meV range. Due to the low energies, the detection of relic neutrinos has not been possible yet.

- **Solar neutrinos** are observed in an energy range of \( \sim MeV \). By observing solar neutrinos, the standard solar model can be tested and neutrino oscillations could be observed [Sup98, SNO02].

- **Galactic neutrino sources** are supernovae and SN remnants as well as strong \( X- \) and \( \gamma- \)ray sources (e.g. \( X- \)ray binaries and neutron stars). Neutrinos can also be produced indirectly through interactions of the charged CRs with the interstellar medium in the galactic plane. Other possible galactic sources are microquasars which are compact radio objects with a stellar black hole. The objects have a similar appearance as quasars, but have a less massive black hole.

- **Extragalactic neutrino sources** are able to accelerate protons to high energies which produce neutrinos in \( p \gamma \) and \( pp \) interactions as described previously. For instance, particles in Active Galactic Nuclei are assumed to reach high energies. Another possibility is the production of neutrinos in Gamma Ray Bursts (GRBs): These
Figure 1.6: Atmospheric neutrino flux as a function of the neutrino energy. The steep spectrum is due to decaying pions and kaons resulting from nucleon-nucleon interactions in the atmosphere (solid lines: muon spectrum, dashed lines: electron spectrum; upper line for each flavor: horizontal flux, lower line: vertical flux). The dotted lines indicate the spectrum from muon neutrino production by nucleon-nucleon induced charm baryons.

emit a lot of energy in the form of $\gamma$ radiation on short scales\(^6\) possibly from compact sources. The observation of a Supernova in 2003 gives strong indications that GRB result from heavy, optical blue SN explosions. Gamma radiation is emitted in two opposite jets and SN are seen as a GRB if one jet points towards Earth [H\(^+\)03]. Topological Defects (TDs), relics from the time shortly after the Big Bang, can also be sources for neutrinos [Sig98]. These could be magnetic monopoles and domain walls that were generated in the time after the Big Bang when three forces were still unified\(^7\). If there are TDs still remaining, these would decay hadronically so that they will finally produce photons and neutrinos.

Extraterrestrial neutrinos can presumably be observed with neutrino telescopes at high energies at which the atmospheric spectrum is suppressed.

\(^6\)A burst can endure between seconds and hours.

\(^7\)The theories taking into account the unification of the three forces (excluding the gravitation) are called Grand Unified Theories - GUT.
1.3 Active Galactic Nuclei (AGN)

Active Galactic Nuclei are galaxies with a supermassive black hole in the center and an accretion disk. Perpendicular to the disk, there are two relativistic jets presumably producing ultra high energy particles up to $10^{21}$ eV [KRB08]. AGN are found at distances as far as $z \approx 6.4$ [WM03]. Observing distant objects always implies looking back in time. Therefore, AGN carry information about an early stage of the Universe. Active Galactic Nuclei are assumed to accelerate particles up to very high energies. The estimation of the neutrino flux from AGN is important for experimental analysis. In this section, AGN will be classified by their features and a unified model for all AGN classes will be given.

![Ground-based Optical/Radio Image and HST Image of a Gas and Dust Disk](image.png)

Figure 1.7: A radio image (jet), overlapping an optical image (disk) of the AGN NGC-4261 is shown on the right side. It has a width of 88000 ly and the two jets around a luminous core are noticeable. The right image has been taken by Hubble Space Telescope (HST) with a width of $\approx 1250$ ly. The torus around the core can clearly be seen.

1.3.1 Basic features

Looking at the discovery history of Active Galactic Nuclei, the first objects classified as AGN were Seyfert galaxies. These are spiral galaxies with a very bright nucleus, first detected by Carl Seyfert in 1943. Bit by bit, other types of galaxies with bright cores and a strong radio appearance were discovered. Today, these differently apparent galaxies are believed to represent the same type of galaxy only seen from different angles. The unified model for all these galaxies with a bright core is basically the following: AGN are described today as objects with a central engine, which is presumed to be a black hole. An accretion disk forms around the rotating black hole. Furthermore, there are two jets perpendicular to the disk [CO96]. Figure 1.7 shows an overlap of a radio and
an optical image of NGC-4261. The jets are seen at radio wavelength while the core of
the galaxy emit UV and optical light. The magnification of the AGN core is an image
taken with the Hubble Space Telescope. The torus can be seen as well as the accretion
disk as a light spot in the center.

So far, the physics of the two jets is not completely understood. AGN are a possible
source of UHE particles because of the relativistic jet which can produce enormous
particle energies assuming first order diffuse Fermi acceleration (see section 1.1.2).

1.3.2 Classification criteria

Several aspects are considered when classifying different types of AGN: There is a sig-
nificant non-thermal component in each AGN spectrum which follows a power law
(\( \sim \nu^{-\alpha} \)). A power law indicates synchrotron radiation as the dominant radiation pro-
cess. The photons may be boosted to higher energies by inverse Compton scattering.
AGN classes are differentiated by their spectral index at 0.178 GHz. \( \alpha > 0.5 \) means
“steep” and \( \alpha < 0.5 \) indicates “flat”. Spectra with a negative index are called “inverse”.

Another distinction criterion is the power emitted at radio frequencies. All AGN have a
very high radio luminosity compared to normal galaxies. Nevertheless, there are AGN
which are less powerful (“radio-quiet”) than others (“radio-loud”).

Since there are gas clouds around AGN absorbing photons coming from the AGN, the
emission lines (of the gas) are broadened due to the Doppler effect: Depending on the
velocity of the particles in the absorbing gas, the emission lines are narrow (cold gas)
or broad (hot gas). AGN are differentiated with respect to whether they have broad
emission lines (higher velocity) or narrow emission lines (lower velocities). Finally,
AGN occur as spiral galaxies and also as elliptical galaxies. Typically, radio-loud AGN
appear as ellipticals while radio-quiet AGN are spiral galaxies.

1.3.3 AGN classes

The different types of galaxies - all classified as AGN because of their bright, compact
core and their high luminosity in the radio - are subdivided to the following smaller
classes.

- **Seyfert** galaxies were the first objects observed with a very bright nucleus almost
  stellar in appearance (Carl Seyfert, 1943). They are subdivided into galaxies
  with both broad (\( v \approx 1000 - 5000 \text{ km/s} \)) and narrow (\( v \approx 500 \text{ km/s} \)) emission
  lines (Seyfert 1) and those with only narrow emission lines (Seyfert 2). Seyfert 1
galaxies have very broad emission lines for allowed lines (such as H I, He I, He II)
and narrower for forbidden lines (O III). There are a few spectra showing both
broad and narrow allowed lines which are classified as Seyfert 1.5. Seyferts are
radio-quiet compared to other types of AGN. The X-ray emission for Seyfert 1 and
1.5 is strong and very variable: The periods of one cycle vary between hours and

\[ ^{8}\text{Note that some authors define the spectral index with a negative sign, } \nu^{-\alpha}. \text{ However, during this } \]

thesis, the index is taken to be positive as defined above.

\[ ^{9}\text{Forbidden means a very low probability.} \]
days. On the other hand, X-ray emission for Seyfert 2 galaxies is not measured for most of these objects. If resolvable, Seyferts are found to be spiral galaxies in at least 90% of all cases. Also, Seyferts are often accompanied by other galaxies, probably gravitationally interacting [CO96].

- Quasars: Radio telescopes in the late 1950s discovered strong radio sources with starlike appearances and unique spectra. These objects were later classified as quasi-stellar radio sources (quasars). They show emission lines which can only be identified with those in a resting frame by assuming a very high redshift\(^\text{10}\). Subsequently quasars are the most distant galaxies detected so far. Most of the quasars are radio-loud, but there are a few radio-quiet quasars, from now on called “Quasi Stellar Objects” - QSOs. Further subclasses are SSRQs (“Steep Spectrum Radio Quasars”), which have strong emission lines with extended radio emission and a steep spectrum ($\alpha > 0.5$), and FSQRs (“Flat Spectrum Radio Quasars”) with compact radio emission and a flat spectrum ($\alpha < 0.5$).

- Radio Galaxies are radio-loud galaxies which appear with narrow emission lines (Narrow Line Radio Galaxies, NLRGs) and with broad emission lines (Broad Line Radio Galaxies, BLRGs). Radio galaxies are (giant) ellipticals with extended radio emission and a steep spectrum ($\alpha \approx 0.8$). A radio galaxy can appear with extended radio lobes or radiate its energy from a compact core and a halo which is about the size of a normal (visible) galaxy or larger.

- Blazars are distant galaxies which vary rapidly in brightness and which have a high degree of polarization. 90% of all resolved blazars are ellipticals. The most well-known object of this class is BL Lacertae in the northern constellation of Lacerta\(^\text{11}\). Objects with properties similar to BL Lacertae form a blazar subclass called BL Lac. BL Lac objects are characterized by their rapid time variability: In only 24 hours, their luminosities can change by as much as 30%. Furthermore, BL Lac spectra are highly polarized (that means 30 - 40% linearly polarized light) and the spectra barely show emission lines: Observations of the very rare and faint emission lines of BL Lac objects have shown that these lines are highly redshifted which means that BL Lacs are found at cosmological distances [CO96]. Another blazar-subclass are optically violent variable quasars (OVVs). They differ from BL Lacs by their higher luminosity and their broad emission lines. Blazars are assumed to be AGN having their jets pointing directly towards Earth. In that case, a flat spectrum is received, since all spectra overlap.

Table 1.2 below shows different types of AGN classified by their radio activity, the width of the emission lines and the spectral index of their spectrum.

---

\(^{10}\)For example, quasar 3C 273 shows broad emission lines with the pattern of the Balmer series of hydrogen redshifted by $z = 0.158$ which implies a velocity of $v = 0.146 \cdot c$ [CO96]!

\(^{11}\)This is the Latin word for lizard.
<table>
<thead>
<tr>
<th>Type of AGN</th>
<th>Radio</th>
<th>galaxy type</th>
<th>emission lines</th>
<th>spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seyfert 1</td>
<td>quiet</td>
<td>spiral</td>
<td>broad and narrow</td>
<td></td>
</tr>
<tr>
<td>Seyfert 2</td>
<td>quiet</td>
<td>spiral</td>
<td>narrow</td>
<td></td>
</tr>
<tr>
<td>Quasar</td>
<td>loud</td>
<td>elliptical</td>
<td>broad</td>
<td>flat (FSRQ) and steep (SSRQ)</td>
</tr>
<tr>
<td>QSO</td>
<td>quiet</td>
<td>spiral</td>
<td>narrow (BL Lac)</td>
<td>flat or inverse</td>
</tr>
<tr>
<td>Blazar</td>
<td>loud</td>
<td>elliptical</td>
<td>narrow (NLRG)</td>
<td>steep</td>
</tr>
<tr>
<td>Radio Galaxy</td>
<td>loud</td>
<td>elliptical</td>
<td>broad (BLRG)</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.2: The AGN zoo

1.3.4 Morphology

Looking at the radio appearance of AGN, there are two morphological classes: Compact sources with a flat radio spectrum and extended sources with a steep spectrum. In this context, extended emission means that the radio emission region near the core is spatially resolvable. For different regions near the core, different power law spectra are found. So called compact objects are hardly resolvable in the inner region and thus the frequency spectrum of the core which can be seen is actually the mean value of all spectra of the different regions of the AGN’s inner part.

Extended sources vary strongly within their morphology: There are diffuse sources as well as structured sources with two or more luminosity centers in the jets. Furthermore, “double sources” are found, where two bright radio clouds in the two jets are symmetrical to a less luminous core. These radio clouds are called knots.

In 1974, Fanaroff and Riley classified extended radio sources by their morphology [FR74]: Defining \( Q \) as the ratio between the distance of the luminosity centers\(^{12} \) and the total dimension of the source, Fanaroff and Riley divide AGN sources into two classes: They found a strong correlation between the value of \( Q \) and the absolute radio luminosity at a frequency of \( \nu = 0.178 \) GHz and therefore classify AGN by their radio luminosity:

- **FR-I galaxies** are sources with a luminosity of

\[
F_{0.178} \leq 2.5 \cdot 10^{26} \text{ W/Hz}
\]

And hence \( Q < 0.5 \). The radio luminosity decreases with the distance from the center. Knots and jets are observed in FR-I galaxies and the knots are located inside the galaxy. FR-I galaxies have a very complex structure and are often found in dense galaxy clusters.

- **FR-II galaxies** have an absolute radio luminosity of

\[
F_{0.178} \geq 2.5 \cdot 10^{26} \text{ W/Hz},
\]

that is for \( Q > 0.5 \). The size of the luminosity centers range from 100 kpc up to several Mpc. The jets and the knots appear much brighter in FR-II galaxies.

\(^{12}\)Without paying attention to the central source.
1.3. Active Galactic Nuclei (AGN)

which is why knots in FR-II galaxies are called hot spots. These hot spots are
significantly brighter than the central source and in contrast to the knots in FR-I
galaxies, FR-II hot spots are located outside the galaxy with a size of several kpc.
FR-II galaxies appear mainly outside or at the edge of galaxy clusters.

Hot spots emit a broad electromagnetic spectrum, reaching from radio to optical emis-
Since the emission is synchrotron radiation, diffuse shock acceleration can be
assumed (see section 1.1.2). This is the reason why AGN are likely to produce ultra
high energy particles which contribute to the cosmic ray flux beyond the ankle.

1.3.5 Unification

The unified model predicts that all active galaxies classified and described previously
in section 1.3.3 belong to the same type of object. The appearance is different due to
different orientations of the AGN as viewed from Earth and also because of different
rates of accretion and masses of the central black hole. Figure 1.8 shows how the
unification can be represented. All active galaxies seem to be driven by a central engine
which is a rotating, supermassive black hole. Furthermore, matter is accreting around
the black hole, presumably producing a magnetic field which can accelerate particles
away from the galaxy. The two preferential directions are perpendicular to the disk
and two relativistic particle jets can be observed. In the direction of the jets, there
are minor gas clouds and radio lobes. Around the center there is an optically opaque
torus and, closer to the center, there is a region of gas clouds emitting broad lines, the
so called Broad Line Region [ZB02]. The unified model is a simple model and not all
details taking into account AGN can be explained by the unification so far, but there
are some key pieces of evidence in support of it.

- Studies of the $H_\alpha$-line of different AGN show that the $H_\alpha$ luminosity is propor-
tional to the featureless continuum at 4800 Å (on a double logarithmic scale, the
slope is $\sim 1.05$) [Shu81]. This indicates a common origin for H - lines (both broad
and narrow) in Seyfert 1, 2, BLRG, NLRG, quasars and QSOs.

- In 1985, R. Antonucci and J. Miller observed NGC 1068, a Seyfert 2 galaxy, in
polarized light and found a Seyfert 1 spectrum with broad emission lines. This
implies that the Seyfert 1 nucleus is hidden from the direct view of Earth by
optically thick material. The Seyfert 1 spectrum is therefore usually diminished
and overwhelmed by the Seyfert 2 spectrum.

- One hint for a central engine is the rapid time variability of the radio luminosity.

1.3.6 Radiation processes

Beneath the (thermal) blackbody spectrum of a regular star or galaxy, there is a sig-
ificant non-thermal component found in AGN spectra. The monochromatic energy flux
$F_\nu$ detected from the non-thermal component of AGN is (with slight deviation) a power
law [CO96]:

\[ F_\nu \propto \nu^{-\alpha} \]
which is a strong indication of synchrotron radiation: This is energy released by electrons. The power law spectrum is the result of the integration over all single electron spectra as shown in figure 1.9. However, the spectrum bends at a certain critical frequency $\nu_c$, because the plasma becomes opaque to its own synchrotron radiation\textsuperscript{13}. For values $\nu \leq 5 \cdot 10^{12}$ Hz, the spectral index is $\alpha < 2.0$. At frequencies $\nu > \nu_c$, the spectrum becomes as steep as $\alpha \approx 2.5$. This is only a simple ansatz for AGN spectra, since there are other interacting processes. From the spectra it can be concluded that there are obviously no atomic absorption and emission processes which means that the radiating material is completely ionized. Possible (non-atomic) processes are bremsstrahlung, inverse Compton scattering and electromagnetic cascades (induced by high energy $pp$ or $\gamma p$ interactions). A power law spectrum is a good approximation because synchrotron radiation dominates over all other radiation processes and the measured spectra are almost pure power law spectra. Figure 1.10 shows a sketch of a typical AGN spectrum.

The thermal component (blackbody radiation) is seen in the form of the blue bump, also called UV bump, at around $7 \cdot 10^{14}$ Hz $< \nu < 3 \cdot 10^{15}$ Hz. It is likely produced by processes in the disk. The radiation from the UV bump is also referred to as Eddington Radiation.

\textsuperscript{13}This process is called synchrotron self absorption.
1.3. Active Galactic Nuclei (AGN)

Figure 1.9: Synchrotron radiation leads to a power law spectrum as shown in this figure since the single electron spectra overlap and the integral distribution is a power law [CO96].

Figure 1.10: Scheme of the continuum of a typical AGN spectrum [CO96].
1.3.7 The proton spectrum

Assuming Fermi acceleration of relativistic charged particles, the spectrum in an AGN jet can be considered to follow a power law:

$$\frac{dN}{dE_p} \propto E^{-p}.\]

Here, the spectra of protons and electrons are equivalent since the particle type does not influence the resulting spectra from shock acceleration. The accelerated particles emit synchrotron radiation. The total power (per unit volume and per unit frequency) radiated is given as

$$P_{\text{tot}}(\nu) \propto \int P(\nu) \frac{dN}{dE_p} dE_p,$$

in which $P(\nu) \propto F(\nu/\nu_{\text{max}})$ is the single particle synchrotron spectrum [RL79]. Here, $\nu_{\text{max}}$ is the critical frequency

$$\nu_{\text{max}} = \frac{3\gamma q B \sin \alpha}{4\pi m_p c},$$

at which the synchrotron spectrum is cut off, see also figure 1.9. $B$ is the magnetic field, $q$ is the charge of the particle, $\alpha$ is the pitch angle, the angle between the magnetic field and the motion of the particle and $m_p$ is the particle mass. The particle’s energy is proportional to the boost factor $\gamma$, and thus equation (1.5) can be written as

$$P_{\text{tot}}(\nu) \propto \int F(\nu/\nu_{\text{max}}) \gamma^{-p} d\gamma,$$

Substituting $x = \nu/\nu_{\text{max}}$ and considering that the critical frequency depends on the boost factor, $\nu_{\text{max}} \propto \gamma^2$, it follows

$$x \propto \gamma^{-2}.$$

Hence, the total radiated power can be written as

$$P_{\text{tot}}(\nu) \propto \nu^{-(p-1)/2} \int F(x)x^{(p-3)/2}dx.$$

The index of the synchrotron spectrum is dependent on the particle index:

$$\alpha = \frac{p - 1}{2}.$$

That implies, if the synchrotron spectrum of a source can be determined, the index of the particle spectrum can be concluded:\[14\]

\[14\]Note that this is valid for all kinds of sources with Fermi accelerated particles, not only for AGN.
1.3.8 Jet-disk symbiosis

There is strong evidence that AGN jets and disk are symbiotic features as it has been worked out by Falcke et al. [FM95, FB95, Fa96]. The idea of the jet-disk symbiosis will be presented here along general lines. The model is based on observations of quasar luminosities at 5 GHz and it is developed for compact radio cores and also for sources with extended emission. The motivation of the model for compact sources will be described in the following.

If the maximum accretion power of a quasar is $Q_{\text{accer}}$, then the so called disk luminosity $L_{\text{disk}}$, describing the blue bump luminosity, is directly correlated to $Q_{\text{accer}}$:

$$Q_{\text{accer}} = q_i \cdot L_{\text{disk}}$$

as well as the total jet power $Q_{\text{jet}}$:

$$Q_{\text{jet}} = q_j \cdot Q_{\text{accer}} \cdot$$

Here, $5\% < q < 30\%$ and $q_j < 1$ are dimensionless fractions. The accretion power is given by the rate of change in the disk mass $\dot{M}_{\text{disk}}$

$$Q_{\text{accer}} = \dot{M}_{\text{disk}} c^2$$

and therefore the mass that is swallowed by the black hole $M_{\text{swallow}}$ is the total power minus the emitted power $Q_{\text{jet}}$ and $L_{\text{disk}}$:

$$M_{\text{swallow}} = (1 - q_j - q_i) \cdot \dot{M}_{\text{disk}} \cdot c^2.$$  \hspace{1cm} \text{(1.9)}

To determine the quasars radio luminosity, synchrotron radiation processes have to be considered. Defining the gravitational radius $R_g$

$$R_g = \frac{G \cdot M_{BH}}{c^2}$$

with $M_{BH}$ as the mass of the black hole and $G$ as the gravitational constant and the relative radial jet coordinate $r_j$:

$$r_j = \frac{R_{jet}}{R_g}$$

with $R_{jet}$ as the radial jet coordinate. The optical depth $\tau$ for electromagnetic radiation through the central axis of the jet is given by

$$\tau = 2 r_j R_g \frac{\kappa_{\text{sync}}}{\sin i}$$  \hspace{1cm} \text{(1.10)}

with $i$ as the inclination angle and $\kappa_{\text{sync}}$ as a synchrotron radiation parameter as determined in [RL79]. Observing compact cores with an optical depth of $\tau = 1$, the part of the jet where synchrotron self absorption starts (for an observed frequency $\nu_{\text{s,obs}}$), $Z_{\text{sso}}$ is given by

$$Z_{\text{sso}} = 19 \text{ pc} \cdot \left( \frac{x_c^{1/2} \beta_j}{u_3 \cdot \gamma_{j,5}^{-1} \cdot f^{1/2} \cdot \sin i} \right)^{1/3} \frac{GHz}{\nu_{s,obs}/D} \left( q_j / L_{16} \right)^2 \hspace{1cm} \text{(1.11)}$$
Table 1.3: Parameters needed for the jet-disk symbiosis model [Fal96]

<table>
<thead>
<tr>
<th>parameter</th>
<th>explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{j,0} = 6 \pm 2$</td>
<td>Lorentz boost factor</td>
</tr>
<tr>
<td>$\beta_{j,0} = 0.972 \pm 0.019$</td>
<td>velocity: $1 - \frac{1}{\gamma_{j,0}}$</td>
</tr>
<tr>
<td>$i = \frac{1}{\gamma_{j,0}} = \frac{6}{18} + \frac{1}{18}$</td>
<td>inclination angle</td>
</tr>
<tr>
<td>$v_{\text{obs}} = \frac{1}{\gamma_{j,0}} = \frac{6}{18} + \frac{1}{18}$</td>
<td>angle between observer and jet axis</td>
</tr>
<tr>
<td>$L_{16} = \frac{L_{\text{disk}}}{10^{46}}$</td>
<td>luminosity</td>
</tr>
<tr>
<td>$x_e' = x_e \gamma_{\text{min},e} \approx 100$</td>
<td>$x_e$ : rel. $e^-$ number density (units of total proton density)</td>
</tr>
<tr>
<td>$\gamma_{\text{min},e}$</td>
<td>minimal Lorentz factor of relativistic $e^-$ population</td>
</tr>
<tr>
<td>$D = \frac{1}{\gamma_{j,0}} (1 - \beta_{j,0} \cos (v_{\text{obs}}))^{-1}$</td>
<td>Doppler factor</td>
</tr>
<tr>
<td>$q_{j/1} = \frac{Q_{\text{jet}}}{L_{\text{disk}}} = 0.15^{+0.2}_{-0.1}$</td>
<td>total jet power in units: disk lum.</td>
</tr>
<tr>
<td>$\xi = 0.15^{+0.1}_{-0.15}$</td>
<td>fit parameter</td>
</tr>
<tr>
<td>$u_3$</td>
<td>ratio btw. tot. energy density in the jet and the magn. energy density, divided by a factor of 3</td>
</tr>
</tbody>
</table>

The parameters are explained in table 1.3. Thus considering the vertical jet coordinate $Z_j$ normalized to the gravitational radius ($z_j := Z_j/R_g$), the total monochromatic luminosity per frequency interval can be written as

$$L_{\nu_s} = 4\pi \cdot \int_{Z_{\text{min}}}^{\infty} \epsilon_{\text{sync}} \cdot \pi \cdot r_j^2 (z_j) \cdot dz_j.$$  \hspace{1cm} (1.12)

with $\epsilon_{\text{sync}}$ as the synchrotron efficiency [RL79]. To get an estimation of the observed fluxes, the quantities have to be transformed into the observer’s frame:

$$L_{\nu_s,\text{obs}} (\nu_{\text{obs}}) = D^2 \cdot L_{\nu} (\nu_{\text{obs}}/D)$$  \hspace{1cm} (1.13)

$$\nu_s = \frac{\nu_{s,\text{obs}}}{D}$$  \hspace{1cm} (1.14)

$$\sin i = D \sin i_{\text{obs}}.$$  \hspace{1cm} (1.15)

The resulting luminosity per frequency interval is therefore

$$L_{\nu_s,\text{obs}} = 4.5 \cdot 10^{31} \frac{\text{erg}}{s \cdot Hz} \left(q_{j/1} L_{16}\right)^{12} \cdot \frac{D^{13/6} \sin i_{\text{obs}}^{1/6} f_{1/12} x_e^{11/12}}{\sqrt{u_3} \gamma_{j,5}^{11/6} \beta_j^{3/12}}.$$  \hspace{1cm} (1.16)

The conclusions which can be drawn from the jet-disk model are

- Radio and disk are strongly correlated for quasars. That underlines the fact that jet and disk are symbiotic features.
- At a given luminosity, radio-loud quasar cores are brighter than those of radio-weak quasars.
- At disk luminosities of $L_{\text{disk}} \geq 10^{46}$ erg/s, radio-loud quasars can appear with a FR-II type jet. Therefore, quasars of luminosities $L_{\text{disk}} \geq 10^{46}$ erg/s can be identified with FR-II galaxies.
• The power of the jet is approximately $1/3$ of the disk luminosity:

$$Q_{\text{jet}} \approx \frac{1}{3} L_{\text{disk}}.$$  \hspace{1cm} (1.17)

Therefore, the jet is energetically important for disk structure and evolution.

• The model which has been developed for compact sources works for extended sources as well and it will be used in section 2.2.

For a more detailed discussion of the model, see [FMB95, FB95].

### 1.3.9 Neutrino production in AGN

Gamma radiation from AGN has been detected by experiments such as Whipple, HEGRA and CANGAROO [CW99, HEG02, MC01] which observe photons at energies $E > 1$ TeV. Future experiments, aiming a lower energy limit $E > 20 - 50$ GeV are HESS and MAGIC [Ste00, PM99]. A matter of particular interest is the observation of photon emission by EGRET [L+99b] which shows that about half of the high energy sources are AGN. This is surprising because objects in the galactic disk should appear much brighter since they are closer to Earth. Another possibility in the near future is the detection of AGN neutrinos with detectors such as AMANDA, IceCube, ANTARES, Auger, Baikal [CA02, Mon03, B+02, DB02] and more (see also chapter 4).

![Diagram of jet and accretion disk](image_url)

**Figure 1.11:** Possible blueprint for the production of high energy neutrinos and photons. Electrons and protons are accelerated in knots moving along the jet, interacting with photons from the accretion disk [Hal98].
Neutrinos in AGN are believed to be produced via photo-meson production as explained in section 1.2.1. In the hadronic acceleration model, protons and neutrons interact with photons to produce mesons which again produce electron- and muon-neutrinos. The hot spots in the jet are moving with relativistic speed, having a Lorentz boost factor of $\gamma = 10 - 100$. The nucleons may therefore be accelerated via diffuse shock acceleration and interacting with the photons from the blue bump that originate in the disk. That is why ultra high energy particles from AGN are believed to be produced in the inner jet, in the vicinity of the disk (see figure 1.11).

Another possibility for neutrino production is proton acceleration in the shock fronts of the jet. In this model, nucleons are accelerated in these shock fronts and interact with each other or with photons from the jet. According to Falcke et al., the jet power $Q_{jet}$ is a fraction of the disk luminosity $L_{disk}$ (see section 1.3.8) [FMB95]:

$$Q_{jet} \approx \frac{1}{3} L_{disk}.$$ 

Neutrinos produced in nucleon-nucleon or nucleon-photon interactions in the jet carry a fraction $x' < 1$ of the jet power:

$$L_\nu = x' \cdot Q_{jet} = \frac{x'}{3} \cdot L_{disk} =: x \cdot L_{disk}.$$ 

$x$ is given by the number of neutrinos $\xi_{kin}$ produced in the interaction relative to other produced particles (this is approximately $\xi_{kin} \approx 1/2$) and the efficiency of the process, $\epsilon_{nth}$. The thermal component of the radiation is approximately 90% and this leads to an efficiency of the non thermal contribution of

$$\epsilon_{nth} \approx 10\%.$$ 

The opacity $\tau$ of $N\gamma$ and $NN$ interactions is basically unknown. As a lower limit,

$$\tau_{N\gamma} \approx \tau_{NN} \approx 1.$$ 

will be assumed. The neutrinos carry thus a fraction

$$x = \frac{1}{3} \xi_{kin} \cdot \epsilon_{nth} \cdot \tau \approx \frac{1}{3} \cdot \frac{1}{2} \cdot 0.1 = \frac{1}{60}$$

of the disk power and the neutrino luminosity can be expressed as a fraction of the disk luminosity:

$$L_\nu = \frac{1}{60} \cdot L_{disk}.$$  \hspace{1cm} (1.18)

The question is whether protons are accelerated in the jet (Hadronic model) or if the particle production in AGN is exclusively leptonic (Leptonic model). If there are no hadrons produced in the jet, no neutrinos will be produced by the AGN. Two facts support the hadronic model. The maximum energy which can be reached by photons is much higher in hadronic than in leptonic models. AGN have been observed by HEGRA up to energies of $\sim 20$ TeV. Furthermore, a leptonic model would demand a correlation between X-ray and TeV radiation of AGN due to inverse Compton scattering of the electrons. This correlation has not been observed yet and it is not necessarily present in a hadronic model.
1.3. Active Galactic Nuclei (AGN)

Figure 1.12: Tree scheme for the selection of AGN neutrino source candidates.
1.3.10 AGN as extragalactic neutrino source candidate

To select AGN source candidates for point source analysis, AGN can be classified by their photon luminosity. AGN have until today only been seen in the photon light, e.g. at radio or optical wavelength as well as in the GeV and TeV range. However, some AGN features might indicate a relatively strong neutrino emission. Figure 1.12 shows a tree scheme of the photon based classification of AGN. Typically, radio loud AGN sources are believed to serve as neutrino candidate sources\textsuperscript{15}. Here, a further subdivision into FR-I and FR-II galaxies is made. These classes can again be subdivided into flat spectrum sources (FSRQ and BL Lac) and steep spectrum radio quasars (SSRQ and FR-I radio galaxies). The flat spectrum sources are also called blazars. This class is particularly interesting when looking for neutrinos, since blazars point their jet right towards Earth and a strong particle emission can be expected. Possible selection criteria are looking at infrared (IR), keV, GeV or TeV sources. Gamma rays inside the AGN are produced when neutrinos are generated in proton interactions. These photons will cascade down to lower energies if the jet is not transparent for the high energy photons. Which of these sources are most likely to contribute most is still to be examined. Additionally, FR-I and FR-II radio galaxies can be selected from the 3CRR catalogue [LRL83]. Two further source candidates are GHz peaked sources (GPS) and Compact steep spectrum sources (CSS) [O'D98]. GPS sources have a flat or inverted spectrum up to a certain frequency in the GHz range, where the spectrum steepens. The steepening of the spectrum could be explained by assuming that a jet is pointing towards Earth which is stopped in dense matter. The high density can be correlated with a second galaxy, interacting or even merging with the AGN. In a similar vicinity, the compete spectrum can be steepened and the source appears as a CSS. The high matter density gives a good target for proton interactions with a high neutrino production rate.

1.3.11 AGN neutrino spectra predictions

In this section, flux predictions from various types of AGN will be discussed. Figure 1.13 shows the the models which will be discussed in the following. The conventional and charm including atmospheric fluxes are also indicated. The figure shows only muon neutrino fluxes. In case of the AGN models, the muon neutrino flux is equal to the electron neutrino and also to the tau neutrino flux.

Model 1 (BBR) in figure 1.13 is a model for steep spectrum quasars [Bec03]. The generic spectrum is determined using the jet-disk symbiosis model [FMB95, FB95]. Neutrino production is assumed to be in the jets. This model will be extended in chapter 2, where the index of the proton energy spectrum will be calculated from the synchrotron emission. Additionally, a more advanced radio luminosity function (RLF) of the steep spectrum AGN population [W+00] will be used and a flat spectrum population [P ea85] will be examined.

In model 2 (MPR) was developed for pγ interactions in blazar jets [MPR01]. Three assumptions are applied during the calculation.

\textsuperscript{15} For instance, strong radio emission in the GHz region implies synchrotron emission. If the synchrotron emission comes from protons, neutrinos are likely to be produced in pp and pγ interactions.
1.3. Active Galactic Nuclei (AGN)

Figure 1.13: Muon neutrino flux predictions from AGN.. Model 1: \(p \gamma\) interactions; Becker, Biermann and Rhode (BBR); Model 2: \(p \gamma\) interactions, Mannheim, Protheroe, Rachen (MPR); Model 3: Mannheim (M), \(p \gamma\) and \(pp\) in the AGN’s host galaxy; Model 4: Rachen & Biermann (RB), \(p \gamma\); Model 5: Atmospheric charm flux. The dashed lines give the atmospheric flux prediction from light baryon (i.e. no charm contribution) decay (horizontal and vertical) [VZ80, H+95].

(i) Neutrons from photo-hadronic interactions can escape freely from the source.

(ii) Magnetic fields in the Universe do not affect the Cosmic Ray flux.

(iii) The neutron flux from the source can be concluded from the observable Cosmic Ray flux.

Following from the third assumption, the proton spectrum at the source \(Q_p(E_p)\) is a power law with an exponential cutoff:

\[
Q_p(E_p) \propto E_p^{-2} \exp(-E_p/E_{max}) \rightarrow [s^{-1} GeV^{-1}].
\]

Neutrons and pions are produced in \(p \gamma\) interactions and thus the neutron spectrum \(Q_n(E_n)\) can be concluded from the proton spectrum. Since charged pions decay into leptons and neutrinos, the muon neutrino flux \(Q_{\nu}\) is correlated with the neutron spectrum as follows (for exact calculation see [MPR01]):

\[
Q_{\nu} \propto Q_n(25E_\nu).
\]

The observable neutrino background is given by

\[
\frac{dN}{dE_\nu} \propto \int_{z_{max}}^{z_{min}} \frac{(1+z)^2}{4\pi d_L^2} \frac{dV}{dz} dV \langle Q[(1+z)E_\nu, z]\rangle dz,
\]
in which \( dI/dz(\lambda'(1+z)E_\nu,z) \) is the input spectrum per comoving volume \( dV/dz \).

Models 3 (M) is given in [Man95]. \( p\gamma \) interactions in blazar jets and \( pp \) interactions in the host galaxy of the jets are taken into account. The proton injection spectrum is the same as in model 2 (for \( p\gamma \) as well as for \( pp \) interactions). The observable muon neutrino flux in an Einstein-de Sitter cosmology\(^{16}\) is given as

\[
\frac{dN}{dE_\nu} = \frac{c}{4\pi H_0} E_\nu^{-1} \int_{L_{\min}}^{L_{\max}} dL X \int_0^{z_{\max}} dz (1 + z)^{0.06} \frac{dn_0}{dL} \frac{dL}{dL X} dE_\nu^2, \tag{1.21}
\]

where \( dN_0/dL_X \) is the present day X-ray luminosity function of the sources. Here, a broken power law luminosity function by Maccacaro et al. [M+91] is used by Mannheim [Man95].

The single source neutrino spectrum is described by \( dL/dE_\nu = E_\nu dN/dE |_{\nu en} [L_X, E_\nu(1+z)] \).

In case of radio-quiet AGN, the normalization condition of the neutrino spectrum is calculated from the original electromagnetic energy flux assuming that the electromagnetic cascade output explains the total diffuse X-ray background. For radio-loud AGN, it is assumed that the present-day energy flux is equal to the diffuse \( \gamma \) -ray background above \( 100 \) MeV\(^{17}\). Radio-loud AGN produce ultra-luminous \( \gamma \) -rays which are directly connected to the (photo-induced) pions from AGN. Neutrinos from \( p\gamma \) interactions are related to the pion flux and can therefore be determined by using the photon flux.

The normalization of the neutrino spectrum from \( pp \) interactions in the vicinity of the blazar jet requires not to overproduce the diffuse \( \gamma \) -ray background. Model 4 (RB) is based on a calculation made by Rachen and Biermann who compute the CR flux from AGN [RB93]. This model is applied to neutrinos by Protheroe and Johnson [PJ96]. Rachen and Biermann include FR-II radio galaxies in their calculation. In [RB93], the cosmic ray flux from FR-II galaxies fits the observed spectrum at energies of \( 10^{19} - 10^{20} \) eV within a factor of 10. Below these energies, the prediction fits the Fly's Eye data for the proton spectrum [RSB93]. The neutrino spectrum is determined according to equation (1.20). The injection spectrum is considered to be a power law with a spectral index of \(-2\) modified by an exponential cutoff function. An Einstein-de Sitter universe is applied with a Hubble parameter of \( h = 0.75 \). The flat spectrum source population from are considered [Pea85]. Protheroe and Johnsson calculate the neutrino spectrum by following the secondary particles from AGN protons interacting with the 2.7 K \( \gamma \) background.

---

\(^{16}\) \( \Omega_M = 1, \Omega_\Lambda = 0, h = 0.5 \).

\(^{17}\) This assumption is based on results from EGRET, which detected about 70 radio-loud AGN sources.
1.4 Cosmology

When calculating the neutrino flux from very distant AGN it is necessary to consider cosmological effects. Although much progress has been made in determining the cosmological parameters, there are still considerable uncertainties in the measured values. This part gives a brief overview of cosmological models and their parameters.

1.4.1 Cosmological parameters

The Hubble constant $H(t)$ is given by

$$H(t) = \frac{\dot{R}(t)}{R(t)}.$$  \hfill (1.22)

Here, $R(t)$ is the scale factor. The Hubble constant is a measure of the expansion per time interval of the Universe. For historical reasons, $H$ is called a constant, but today there are implications that $H$ is varying with time. The Hubble constant observed today, $H_0$, can be expressed as

$$H_0 = 100h \cdot \frac{km}{s} \frac{1}{Mpc},$$  \hfill (1.23)

where $h$ is the dimensionless Hubble parameter [Wei72]. Measurements of the Cosmic Microwave Background (CMB) restrict the parameter to $[B+03]$:

$$h = 0.71^{+0.04}_{-0.03}.$$  \hfill (1.24)

The deceleration parameter $q(t)$ is the change in the expansion rate, i.e. if the Universe is decelerating $q > 0$ or if it is accelerating $q < 0$:

$$q(t) = -\frac{\ddot{R}(t)R(t)}{R(t)^2}.$$  \hfill (1.25)

The normalized matter density $\Omega_m$ is set to

$$\Omega_m(t) = \frac{8\pi G}{3H(t)^2}\rho(t).$$  \hfill (1.26)

The critical mass density, $\Omega_m = 1$, is given for $\rho = \rho_c = 3H(t)^2/8\pi G$. It is called critical since it gives the critical value between collapse and eternal expansion in a flat Universe ($k = 0$) with $\Omega_\Lambda = 0$.

The normalized cosmological constant $\Omega_\Lambda$ is defined analogously to the normalized matter density:

$$\Omega_\Lambda(t) = \frac{c^2}{3H(t)^2}\Lambda.$$  \hfill (1.27)
The critical cosmological constant, $\Lambda_c$, is defined as the constant which Einstein tried to use to keep the Universe static. A static Universe implies $\ddot{R} = \dot{R} = 0$ which, leads to

$$\Lambda_c = 4\pi G \rho(t)c^2,$$  \hspace{1cm} (1.28)

using the Friedmann equations. However, Einstein finally condemned the idea of a cosmological constant because of the instability of this state. Today, the cosmological constant is known to be $\Omega_\Lambda \neq 0$.

The normalized curvature $\Omega_k$ is defined as

$$\Omega_k = -\frac{c^2k}{\dot{R}(t)^2}. \hspace{1cm} (1.29)$$

The state of the Universe is totally determined by the previously mentioned parameters, since the Einstein equations can now be written as

$$\Omega_\Lambda(t) - \frac{1}{2}\Omega_m(t) = -q(t) \hspace{1cm} (1.30)$$
$$\Omega_m(t) + \Omega_\Lambda(t) + \Omega_k = 1. \hspace{1cm} (1.31)$$

The fundamental cosmological diagram (figure 1.14) shows $\Omega_\Lambda$ vers. $\Omega_m$ and the critical thresholds for the geometry of the Universe as well as expansion and collapse. Several models have been presented in the past, e.g. the Einstein-de Sitter model with $k = 0$, $\Lambda = 0$, $q = 1/2$ which would lead to an eternal expansion. Today, it is well determined that there is a non-zero cosmological constant ($\Omega_\Lambda = 0.75 \pm 0.04$) and a material density $\Omega_m = 0.27 \pm 0.04 \ [S^+03]$. 
Figure 1.14: The cosmological diagram taken from [PS99]. The two lines determine whether the Universe is flat, closed or open and if it will expand eternally or recollapse. Combining the SN data with the CMB data, the Universe is flat ($\Omega_m + \Omega_\Lambda = 1$). An $\Omega_\Lambda = 0$ Universe can be ruled out by SN measurements with a confidence of $P(\Omega_\Lambda > 0) = 99\%$. The upper shaded region represents cosmologies which do not allow a big bang (here, the age of the Universe diverges) and the lower shaded region corresponds to a Universe that is younger than the oldest heavy elements.
1.4.2 Cosmological distances

Looking at an object at a distance \( r_1 \), neither the distance \( r_1 \) nor the lookback time \( t_1 \) is directly measurable. Instead, measurable quantities are introduced. The \textit{angular distance} is defined as

\[
d_{A} = \frac{D}{\theta}
\]  

(1.32)

where \( D \) is the known proper size of the object and \( \theta \) is the angular diameter.

The \textit{proper motion distance} is given by the known velocity \( u \) of an object per apparent angular motion \( \dot{\theta} \):

\[
d_{m} = \frac{u}{\dot{\theta}}.
\]  

(1.33)

The \textit{luminosity distance} \( d_L \) is defined as [CO96]

\[
d_L^2 = \frac{L}{4\pi F}.
\]  

(1.34)

Here, \( F \) is the radiant flux measured for a light emitting source with a luminosity \( L \). The relation between these measurable quantities and theoretical quantities such as the scale factor at present time \( R_0 \) and at the time \( t_1 \) at which the object is seen \( R_1 \) is [RL79]

\[
(1 + z) = \frac{R_0}{R_1} \quad d_A = R_1 \cdot r_1
\]  

(1.35)

\[
d_M = R_0 r_1 \quad d_L = \frac{R_0^2 r_1}{R_1}.
\]  

(1.36)

Thus, the distance measures are connected by the redshift:

\[
d_L = (1 + z) \cdot d_M = (1 + z)^2 \cdot d_A.
\]  

(1.37)

Using the integral presentation of the lookback time (see [CPL92]), the luminosity distance can be written as a function of \( z_1 \):

\[
d_L = \frac{(1 + z) \cdot c}{H_0 \cdot |\Omega_k|^{1/2}} \sin^n \left\{ |\Omega_k|^{1/2} \cdot I(z) \right\}.
\]  

(1.38)

Here, \textit{\textit{\textit{sinn}}} is defined as follows [CPL92]:

\[
sinn = \begin{cases} 
\sin & \text{for } \Omega_k > 0 \\
\sinh & \text{for } \Omega_k < 0 \\
disappears together with both } \Omega_k & \text{for } \Omega_k = 0
\end{cases}
\]  

(1.39)

and \( I(z) \) is an integral

\[
I(z) := \int_0^z \left( (1 + z')^2 \cdot (1 + \Omega_m z') - \Omega_A z' (2 + z') \right)^{-1/2} dz'.
\]  

(1.40)

The integral can be evaluated numerically with the result shown in figure 1.15. In this figure, the three distance measures are compared using the current values, \( \Omega_m = 0.27 \) and \( \Omega_A = 0.75 \).
Figure 1.15: Distance measures versus redshift for $\Omega_m = 0.27$ and $\Omega_\Lambda = 0.75$, taking $\Omega_k \approx 0$ as assumed in [S+03].

1.4.3 The Comoving Volume

When counting objects at cosmological distances, it is common to determine the comoving densities of the objects. To estimate the actual number of galaxies, these densities have to be multiplied with the comoving volume.

The comoving volume element in the Robertson-Walker metric is given by [CPL92]

$$dV_c = \frac{d^3d_M}{(1 + \Omega_k \cdot (H_0/c)^2 \cdot d_M^2)^{1/2}} d(d_M) d\Omega. \quad (1.41)$$

The integral over the solid angle and the motion distance can be evaluated analytically with the result

$$V_c(d_M) = \left\{ \begin{array}{ll}
(\frac{c}{H_0})^3 (2\Omega_k)^{-1} & \left[ H_0/c \cdot d_M (1 + \Omega_k (H_0/c)^2 d_M^2)^{1/2} \right. \\
-\left[ \Omega_k^{-1/2} \sin^{-1}(H_0/c \cdot d_M [\Omega_k^{1/2}]) \right] & \text{for } \Omega_k \neq 0 \\
\frac{4\pi}{3} d_M^3 & \text{for } \Omega_k = 0.
\end{array} \right. \quad (1.42)$$

1.4.4 Cosmic Microwave Background

In 1964, Penzias and Wilson discovered the Cosmic Microwave Background (CMB) with a radio antenna which was originally used to study radio emission from the Galaxy [HH98]. Today it is known that the CMB is thermal blackbody radiation, distributed isotropically in space with a temperature of $T = 2.735$ K [M+90]. This
isotropic radiation goes back to approximately $10^{-6}$ seconds after the Big Bang, when quarks condensed into hadrons. All baryon-antibaryon pairs annihilated to leave behind photons: These photons make up most of the cosmic microwave background today. When the temperature of the Universe had dropped to $T \approx 3000$ K, the free electrons did not have sufficiently high energy to escape the electric field of the protons and thus they were captured by primordial nuclei. This phase is called recombination. Before recombination photons were tightly coupled to the freely moving electrons through Compton scattering. Due to the coupling of the electrons to the primordial nuclei the photons could escape and the Universe became transparent. That implies that the observable Universe begins at $T \approx 3000$ K which corresponds to a redshift $z \approx 1100$ and a time $t \approx 10^6$ years after the Big Bang [Dur01].

To confirm that the CMB spectrum is truly isotropic, very accurate measurements are necessary. The early ground-based measurements still allowed a small anisotropy since the blackbody spectrum was disturbed by fluctuations in the atmosphere. In 1989, the Cosmic Background Explorer (COBE) satellite was launched. COBE was able to measure the intensity of the CMB blackbody spectrum at $T = 1.735$ K without interference from the atmosphere. Although the spectrum is nearly isotropic, a dipole irregularity occurs due to the motion of Earth (and the solar system as well as the galaxy) through the Universe\(^{18}\). Using COBE data it was possible to determine the Earth’s velocity around the sun (29 km/s), the sun’s local velocity in the Milky Way (220 km/s) and the speed of our galaxy in the local cluster (600 km/s). Subtracting this effect as well as the radiation from the galactic disk anisotropies on the $\mu$K scale remain which are due to density fluctuations at the surface of last scattering. These density fluctuations can be traced back to a time at $z \approx 1500$ and conclusions about the very early Universe are possible.

### 1.4.5 Measuring cosmological parameters

Two different methods are used to determine the current cosmological parameters, namely the small scale anisotropy measurements of the CMB [Dur01] and very early type Ia supernovae [PS97].

To measure small scale anisotropies, the multipole spectrum of the temperature fluctuations is observed. Current experiments measure the spectrum up to an order of $l \sim 1500$ [TZ02]. The spectrum can be evaluated theoretically and fitted to the data by varying the cosmological parameters. The best fit parameters are given in table 1.4.

<table>
<thead>
<tr>
<th>$\Omega_b h^2$</th>
<th>$\Omega_{\Lambda}$</th>
<th>$\Omega_m$</th>
<th>$\Omega_{\text{tot}}$</th>
<th>$h$</th>
<th>Age (Gyr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.022$^{+0.009}_{-0.009}$</td>
<td>0.7$^{+0.04}_{-0.04}$</td>
<td>0.2$^{+0.04}_{-0.04}$</td>
<td>1.0$^{+0.02}_{-0.02}$</td>
<td>0.7$^{+0.04}_{-0.04}$</td>
<td>13.7$^{+0.2}_{-0.2}$</td>
</tr>
</tbody>
</table>

Table 1.4: Current cosmological parameters [S+03]. Here, $\Omega_b$ is the baryonic matter density. The results are mean and with 68% confidence errors. Results are fit to the WMAP, CBI, ACBAR, 2dFGRS and Lyman $\alpha$ forest data.

Another method to determine the cosmological parameters ($\Omega_m$, $\Omega_{\Lambda}$ and $h$) is using

---

\(^{18}\)The fluctuations are around a temperature scale of $\Delta T \approx nK$. 

distant supernovae (SNe). Since the type Ia SN can be used as a standard candle, the conversion from luminosity to redshift is possible. The relation between effective magnitude and redshift can be measured and the data can be fitted with different cosmological parameters. The constraints to the cosmological parameters $\{\Omega_m, \Omega_A\}$ are shown in figure 1.14 where the best-fit confidence regions in the $\Omega_m - \Omega_A$ plane are shown (68%, 90%, 95% and 99%). The “Supernova Cosmology Project” [PS99] has taken these data using different telescopes such as the Hubble Space Telescope, Keck and others to observe the sky shortly after the new moon. About three weeks later, the same regions in the sky are observed again in order to compare the pictures. If there is a relevant luminosity peak, this object is likely to be a supernova and the light curve can be taken while still brightening since an average SN has a rise time $> 3$ weeks.
Chapter 2

An Estimation of the Integral Neutrino Flux from AGN

In the following chapter, a detailed description of the calculation for a model of the isotropic neutrino flux from AGN is given. The model includes FR-II sources with strongly extended radio emission as well as flat spectrum blazar sources. In contrast to other models, the spectral index of the proton spectrum will be calculated from the synchrotron emission of the electrons. This is possible since the proton and electron spectra resulting from shock acceleration are equivalent. Further, the maximum energy of the protons in the jets will be assumed to be luminosity dependent as is suggested in [Lov’76]. To normalize the generic AGN energy spectrum, the jet-disk symbiosis model given in [FMB’95] will be applied to connect the neutrino luminosity with the disk luminosity and therefore, also with the radio luminosity of the jets [FMB’95].

In the first section of this chapter, the AGN samples which will be used to determine the AGN density which is also called radio luminosity function. Also, the spectral indices of the sources are used to evaluate the index of the primary particle spectrum. Two different types of AGN samples will be analyzed, one containing steep spectrum sources with extended emission, the second comprising flat spectrum AGN. Section 2.2 comprises an explicit description of the models which are used in the calculation. It gives an overview of the applied normalizations and the fits that have been made. In the third part of the chapter, limits and constraints on the neutrino flux from AGN are discussed. Finally, the different models for both types of AGN sources are applied to calculate the AGN neutrino flux and the results are discussed.

A list of the used samples is given in appendix A and the calculation results are summarized in appendix B.1.

2.1 The samples and their indices

In this section, the index distribution for two different types of AGN samples is analyzed.

2.1.1 The samples

The first sample consists of 356 steep spectrum sources, selected by Willott et al. [W+’00]. The sources are sampled from the 7CRS [MR90, L+’99a], 6CE [REL00] and 3CRR [LRL83] catalogues with flux measurements at 178 MHz. A list of the samples can be found in
2.1. The samples and their indices

table A.1 in appendix A. The second sample comprises 171 flat spectrum sources from Dunlop et al. [Pea85]. The sources are listed in table A.2 in appendix A. They are selected from the Parkes selected regions sample, the Parkes ±4° zone [Pea85, WJB82], the 'Northern-Sky' survey of [PW81] and the 'All-Sky' survey of [WP85].

2.1.2 The spectral and particle index of the sources

The index $p$ of the proton spectrum is correlated to the index $\alpha$ of synchrotron radiation as

$$p = 2\alpha + 1.$$  \hfill (2.1)

- **Steep spectrum sources**

124 of the total 356 sources are identified in the S1-S5 catalogues [P+72, PK72, P+78, K+81] which give the source index at 5 GHz. The indices of the sources are given in table A.1 in appendix A. To estimate the steep spectrum index $\alpha_s$ at 5 GHz, the maximum in a distribution of the source counts against $\alpha_s$ is determined applying a Gauß fit (see figure 2.1). The maximum is found to be at

$$\alpha_s = 0.8$$

with a peak width of $\sigma_s = 0.2$. In calculations of the AGN evolution functions by Willott et al., a spectral index of $\alpha_s = 0.8$ is assumed [W+00]. The spot check of the sources supports this assumption. Hence, the index of 0.8 is adopted. Subsequently the particle index is

$$p_s = 2 \cdot \alpha_s + 1 = 2.6.$$  \hfill (2.2)

- **Flat spectrum sources**

The synchrotron radiation index at 5 GHz for all 171 flat spectrum sources is given in table A.2 in appendix A. The spectral index distribution of the sources is shown in figure 2.2. A Gauß fit has been applied with a maximum at

$$\alpha_f = 0.25$$  \hfill (2.3)

and a width of $\sigma_s = 0.42$. This index is applied in all following calculations taking into account flat spectrum sources. The synchrotron index corresponds to a particle index of

$$p = 2 \cdot \alpha_f + 1 = 1.5$$  \hfill (2.4)

if it is assumed that the proton and the electron spectrum can also be identified in the case of flat spectrum sources. This is an approximation since due to the superposition of the electron spectra in flat spectrum sources, inverse Compton scattering effects can occur.
Figure 2.1: Histogram of the spectral indices of the steep spectrum sources. Sources counts are shown against the spectral index at 5 GHz. The maximum is found at $\alpha_s = 0.8$. The width of the Gaussian distribution is $\sigma_s = 0.2$.

Figure 2.2: The histogram of the flat spectrum spectral indices at 5 GHz. Sources counts are shown against the spectral index. A Gaussian function has been fitted with a maximum at $\alpha_f = 0.25$. 
2.2 Ingredients of the flux calculation

The isotropic neutrino flux depends on four important facts:

1. The generic AGN neutrino energy spectrum, \( \frac{dN}{dE_\nu} \), given per time and energy interval. The spectrum depends on the neutrino energy and on the redshift of the AGN. Additionally, a luminosity dependence will be derived by applying the jet-disk symbiosis model [FMB95].

2. The AGN Radio Luminosity Function (RLF), \( \frac{dn}{dL(z, L)} \), which is a function of the radio luminosity and redshift. The RLF is given per comoving volume and luminosity interval.

3. The comoving volume \( dV/dz \) is used to convert the RLF from a density to a number. Further, the flux decreases with the luminosity distance as \( 1/(4 \pi d_L^2) \).

4. The flux is integrated over luminosity and redshift.

The product of the generic spectrum, of the AGN RLF and of the comoving volume gives the neutrino flux per energy, time and luminosity. Further, it has to be considered that the flux decreases with cosmological distance. With \( d_L \) as the luminosity distance, the flux decreases thus with \( 4\pi d_L^2 \). The result is the flux at a given redshift and a given luminosity. To obtain the flux which is arriving at Earth \( \Phi(E_\nu^0) \), all redshifts and luminosities which occur have to be added up:

\[
\Phi(E_\nu^0) = \int_{\nu} \frac{dN}{dE_\nu} (E_\nu^0, L, z) \cdot \frac{dn}{dL} (L, z) \cdot \frac{dV}{dz} \cdot \frac{1}{4\pi d_L^2}.
\]

(2.5)

The neutrino energy is assumed to evolve with redshift. Thus the energy at the AGN \( E_\nu \) is connected to the energy after a travel distance \( z \), \( E_\nu^0 \), as

\[
E_\nu = (1 + z) \cdot E_\nu^0.
\]

The result of the calculation is the neutrino flux per space, angle and time and energy \( \langle \Phi \rangle = 1/(sr \cdot s \cdot cm^2 \cdot GeV) \). The functions mentioned above will be explained and normalized in the following subsections.

PARAMETERS AND VARIABLES

To simplify the understanding of the calculation, a list of parameters is given below. In following equations, it will be assumed that these parameters are known.

- \( L_{0.151} \): Radio power, taken at the frequency \( \nu = 0.151 \) GHz in units of \([W/Hz/sr]\).
- \( P_{\text{radio}} \): Radio power, taken at the frequency \( \nu = 5 \) GHz, for extended quasars in units of \([\text{erg/s/Hz}]\).
- \( L_{\text{radio}} \): Radio luminosity in units of \( \text{erg/s} \). This is the radio power, integrated over the frequency.
\begin{itemize}
  \item $L_{42}$: Radio luminosity in units of $10^{42}$ erg/s: \( L_{42} = L_{\text{radio}}/(10^{42} \text{ erg/s}) \).
  \item $L_{\text{disk}}$: Luminosity of the blue bump. It is denoted $L_{\text{disk}}$, since the blue bump is produced by photons from the accretion disk.
  \item $E_{\nu}$: Neutrino energy at the AGN.
  \item $E_{\nu}^0$: Neutrino energy at Earth.
  \item $H_0$: Hubble parameter today, \( H_0 = H(t = t_0) \).
  \item $h$: Hubble parameter in units of 100 km/(s \cdot Mpc):
    \[
    H_0 = 100 \cdot h \frac{\text{km}}{\text{s} \cdot \text{Mpc}}.
    \]
  \item $c$: Speed of light in vacuum.
  \item $\Omega_\Lambda$: Vacuum energy density due to the cosmological parameter $\Lambda$.
  \item $\Omega_m$: Matter density normalized to the critical density which gives the critical value between collapse or eternal expansion of the Universe if $\Omega_\Lambda = 0$.
  \item $\Omega_k$: Normalized curvature (see section 1.4).
  \item $d_L$: Luminosity distance.
  \item $\alpha$: Spectral index of the AGN synchrotron spectrum at 5 GHz.
  \item $p$: Spectral index of the AGN proton (neutrino) spectrum.
\end{itemize}

Note that the cosmological parameters are today experimentally well determined (\( \Omega_m = 0.27^{+0.04}_{-0.04} \), \( \Omega_\Lambda = 0.73^{+0.06}_{-0.06} \) and \( h = 0.71^{+0.04}_{-0.03} \), see [S+03] and [PS97]). Inflationary theory implies that the Universe is flat (\( \Omega_k = 0 \)). However, most of the models used are still calculated in the former standard cosmology (\( \Omega_m = 1, \Omega_\Lambda = 0 \)) with a Hubble parameter of \( h = 0.5 \). To be consistent in further calculations, these values will be adopted. In chapter 3, the uncertainties in flux calculations due to the choice of cosmological parameters will be discussed more closely. It will be shown for two sets of parameters that, if the calculation is made with a consistent choice of a set of parameters, the result barely differs.

### 2.2.1 The generic neutrino flux

The generic AGN neutrino flux is adopted to follow the proton spectrum. This can be described as a power law with a spectral index $p$ having an exponential cutoff. This cutoff depends on the particle’s maximum energy:

\[
\frac{dN}{dE_{\nu}} = \phi_{\nu} \left( L_{\text{disk}} \right) \cdot E_{\nu}^{-p} \exp \left( -\frac{E_{\nu}}{E_{\nu}^{\text{max}}} \right) \cdot \exp \left( -\frac{E_{\nu}}{E_{\nu}^{\text{max}}} \right).
\]  

(2.6)

The power law behavior is due to Fermi acceleration of the protons in shock fronts of the jet. The spectrum is limited by the strength of the magnetic field of the source.
This determines the maximum energy of the accelerated proton, there is an exponential cutoff in the spectrum. The magnetic field is connected to the disk luminosity of the AGN and subsequently the maximum proton energy can be expressed in terms of the disk luminosity [Lov76]:

\[ E_p^{\text{max}} \propto \sqrt{L_{\text{disk}}} \cdot \]

The generic spectral index can be derived from the electron synchrotron spectral index of the sources as described in section 1.3.7.

The normalization of the generic neutrino spectrum, \( \phi_0 \), can be found by assuming that particles from an AGN produce the same order of magnitude of luminosity as the electromagnetic spectrum, \( L_{\text{disk}} \), and neutrinos contribute with a fraction \( x < 1 \) [FMB95]. It is assumed that the power of the neutrinos is directly connected to the power of the jet. This in turn is proportional to the disk luminosity which is a fraction \( q = 1/3 \) of the accretion power at high accretion rate. The factor \( x \) results from assuming a kinematic factor of \( \xi_{\text{kin}} \approx 1/2 \) and a (non thermal) efficiency of \( \epsilon_{\text{nth}} \approx 10\% \), see section 1.2.1. The optical depth of AGN sources is basically not known. An opacity of \( p \gamma \) and \( pp \) interactions of \( \tau := \tau_{\gamma \gamma} \approx \tau_{pp} \approx 1 \) is assumed. This can be used as a lower limit and an estimation of a maximal optical depth is possible when comparing current neutrino flux limits to the calculated flux. The fraction \( x \) is

\[ x = q \cdot \tau \cdot \xi_{\text{kin}} \cdot \epsilon_{\text{nth}} \approx \frac{1}{60} \cdot \]

Thus the neutrino luminosity from the jets is given as a fraction of the disk luminosity:

\[ \int_{E_{\nu}^{\text{min}}}^{E_{\nu}^{\text{max}}} E_{\nu} \frac{dN}{dE_{\nu}} \cdot dE_{\nu} = \int_{E_{\nu}^{\text{min}}}^{E_{\nu}^{\text{max}}} E_{\nu} \cdot \phi_0 \cdot E_{\nu}^{-p} \exp \left( -\frac{E_{\nu}}{E_{\nu}^{\text{max}}} \right) dE_{\nu} = x \cdot L_{\text{disk}} \cdot \quad (2.7) \]

- \( p = 2 \):
  Provided that \( p = 2 \) the integral can be evaluated by developing the exponential function in a Taylor series and interchanging the integral with the sum. The result is then

\[ x \cdot L_{\text{disk}} = \phi_0 \cdot \left[ \ln \left( \frac{E_{\nu}^{\text{max}}}{E_{\nu}^{\text{min}}} \right) + \sum_{n=1}^{\infty} \frac{1}{n \cdot n!} (-1)^n \sum_{n=1}^{\infty} \frac{1}{n \cdot n!} \left( -\frac{E_{\nu}^{\text{min}}}{E_{\nu}^{\text{max}}} \right)^n \right] \cdot (2.8) \]

The lower integration limit is given by the rest mass of the pion, \( E_{\nu}^{\text{min}} = E_{\pi^\pm} = 139.57 \text{ MeV} \). This is the energy which is at least necessary to induce the \( p \gamma \) or \( pp \) interaction. The upper integration limit is assumed to be varying with the disk luminosity, see equation 2.2.1. Since the maximum energy is much bigger than the minimum neutrino energy, \( E_{\nu}^{\text{max}} > E_{\nu}^{\text{min}} \), the last term in equation 2.8 is approximately zero:

\[ \sum_{n=1}^{\infty} \frac{1}{n \cdot n!} \left( -\frac{E_{\nu}^{\text{min}}}{E_{\nu}^{\text{max}}} \right)^n \approx 0. \]
The second term can be approximated as

\[
\sum_{n=1}^{\infty} \frac{1}{n \cdot n!} (-1)^n \approx -1 + \frac{1}{4} \cdot \frac{1}{18} + \frac{1}{96} \ldots \approx -0.795.
\]

The normalization constant for the generic spectrum can now be determined to be

\[
\phi_0 = \frac{x \cdot L_{\text{disk}}}{\ln \left( \frac{E_{\text{max}}}{E_{\text{min}}} \right) - 0.795}
\]  

(2.9)

Regard that \( L_{\text{disk}} \) is still given in units of erg/s and has to be converted to GeV/s for the consistency of the units.

- \( p \neq 2 \):
  - If \( p \neq 2 \), the integral is solved as follows:

\[
x \cdot L_{\text{disk}} = \phi_0 \cdot \int \left( \frac{E_\nu}{\text{GeV}} \right)^{-p+1} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left( \frac{E_\nu}{E_{\text{max}}} \right)^n d(E_\nu/\text{GeV})
\]

\[
= \phi_0 \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left( \frac{E_\nu}{E_{\text{max}}} \right)^n \int \left( \frac{E_\nu}{\text{GeV}} \right)^{-p+n+1} d(E_\nu/\text{GeV})
\]

For \( p \notin \mathbb{N} \setminus \{1\} \), the integral can be evaluated to be

\[
x \cdot L_{\text{disk}} = \phi_0 \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left( \frac{E_\nu}{E_{\text{max}}} \right)^n \cdot \left[ \frac{1}{-p + n + 2} \left( \frac{E_\nu}{\text{GeV}} \right)^{-p+n+2} \right] E_{\text{max}}^{E_{\text{max}}}.
\]

In the following, two cases will have to be discussed. In case of steep spectrum sources, the spectral index of the neutrino spectrum is between 2 < \( p < 3 \). The synchrotron radiation index is \( \alpha > 0.5 \) and in most cases \( \alpha < 1 \). It is connected to the particle index as described in equation 2.1. For flat spectrum sources, the synchrotron index is given as \( \alpha < 0.5 \) and thus the particle index is \( p < 2 \).

\( \circ \) **Steep spectrum sources** \( (2 < p < 3) \)

Due to the allowed range of the particle index, the integration limits can be inserted with the approximated result

\[
x \cdot L_{\text{disk}} \approx \phi_0 \left\{ \frac{1}{p-2} \left( \frac{E_{\text{min}}}{\text{GeV}} \right)^{2-p} - \left( \frac{E_\nu}{\text{GeV}} \right)^{2-p} \right\} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!(-p+n+2)} \left[ \frac{E_{\text{max}}}{\text{GeV}} \right]^{2-p}.
\]

It has been taken into account that the lower integration limit can be neglected in comparison to the upper limit for summation indices \( n \geq 1 \). For the first term, \( n = 0 \), the power of the energies is still negative, so that the lower limit dominates over the upper limit. The remaining series can be approximated since the contribution of the addends decrease with increasing summation index. The approximated result is

\[
x \cdot L_{\text{disk}} \approx \phi_0 \left\{ \frac{1}{p-2} \left[ \frac{E_{\text{min}}}{\text{GeV}} \right]^{2-p} + f(p) \cdot \left[ \frac{E_{\text{max}}}{\text{GeV}} \right]^{2-p} \right\},
\]
with

$$f(p) = \frac{1}{2 - p} - \frac{1}{3 - p} + \frac{1}{2} \cdot \frac{1}{4 - p} - \frac{1}{6} \cdot \frac{1}{5 - p} + \frac{1}{24} \cdot \frac{1}{6 - p}. \quad (2.10)$$

For steep spectrum sources, the normalization constant is hence

$$\phi_0^s = \frac{x \cdot L_{\text{disk}}}{\frac{1}{p-2} \left[ \frac{E_{\text{min}}^\nu}{\text{GeV}} \right]^{2-p} + f(p) \cdot \left[ \frac{E_{\text{max}}^\nu}{\text{GeV}} \right]^{2-p}}. \quad (2.11)$$

**Flat spectrum sources** ($p < 2$)

In case of a flat spectrum, the terms of the lower integration limit, $E_{\text{min}}^\nu$ are negligible in all addends since the power of the energy is always positive. Hence, the integral can be approximated as follows

$$x \cdot L_{\text{disk}} \approx \phi_0^f f(p) \cdot \left[ \frac{E_{\text{max}}^\nu}{\text{GeV}} \right]^{2-p},$$

with $f(p)$ as given in eq. (2.10). Therefore, the normalization constant is given as

$$\phi_0^f = \frac{x \cdot L_{\text{disk}}}{f(p) \cdot \left[ \frac{E_{\text{max}}^\nu}{\text{GeV}} \right]^{2-p}}. \quad (2.12)$$

**Jet-disk symbiosis** To convert the disk luminosity into the radio luminosity of the jets, the jet-disk symbiosis model by Falcke et al. is used [FMB95].

**Flat spectrum sources**

The details of the jet-disk symbiosis model for compact cores are explained in section 1.3.8. This model is compared to the data [FB95] and can here be used for the flat spectrum sample. Assuming a power law slope of $\alpha = 0$ the luminosity of the jet in units of erg/s, $L_{\text{radio}}^{\text{comp}}$ is given as

$$L_{\text{radio}}^{\text{comp}} = 6.7 \cdot 10^{42} \frac{\text{erg}}{\text{s}} \cdot D^{2.17} \cdot \sin^{0.17} (i_{\text{obs}}) \cdot (x_c')^{0.83} \cdot \left( \frac{6}{\gamma_j} \right)^{1.8} q_j^{1.42} L_4^{1.42} \cdot \xi. \quad (2.13)$$

The parameters from above are explained in the parameter list below. The correlation between disk luminosity and radio luminosity in units of $10^{42}$ erg/s, $L_{42}^{\text{comp}}$, is

$$L_{\text{disk}} = 2.1 \cdot 10^{45} (L_{42}^{\text{comp}})^{0.79} \left[ \frac{\text{erg}}{\text{s}} \right]. \quad (2.14)$$

**Steep spectrum sources**

The model for extended, optically thin sources will additionally be used for the calculation of the steep spectrum source flux. The relation between the differential radio luminosity at 5 GHz of the lobes and the disk luminosity is

$$P_{\text{lobe}} = 1.8 \cdot 10^{34} \frac{\text{erg}}{\text{s} \cdot \text{Hz}} \left( \frac{\text{GHz}}{\nu} \right)^{0.5} \beta_j^{1/4} \cdot P_\nu^{1/4} \cdot x_{e,100}^{1/4} \cdot \left( \frac{q_j}{L_4} \right)^{3/2}. \quad (2.15)$$
This result is based on the investigation of a quasar sample, described in detail in [FB95]. Assuming a spectral index of $\alpha \approx 0$, the disk luminosity is connected to the integral radio luminosity, $L_{\text{radio}}^e$, as

$$L_{\text{disk}} = 4.29 \cdot 10^{45} \cdot (1 + z)^{-1/2} (L_{42}^e)^{2/3}. \quad (2.16)$$

The parameters which occur in equations 2.15 and 2.13 are given below.

**PARAMETER LIST**

- $\beta_j$: Jet velocity in units of $c$ which can be assumed to have a value of $\beta_j \approx 1$.
- $\gamma_j$: Jet’s Lorentz Boost factor, $\gamma_j = 6$.
- $P_{-12}$: Pressure parameter. The external radiation pressure at the AGN is given by $P_{\text{ext}} = P_{-12} \cdot 10^{-12}$ erg/cm$^3$. The pressure evolves with the time due to the expansion of the Universe. The pressure parameter depends on the redshift as $P_{-12} = (1 + z)^3 \cdot P_{-12}^0$, where $P_{-12}^0$ is the pressure of the Universe at $z = 0$. $P_{-12}^0$ is set to $P_{-12}^0 = 1$.
- $x_{e,100}'$ Modified electron density. Let the ratio between the relativistic electron density and the total number density of protons be $x_e$ and the minimum Lorentz factor of the relativistic electron population divided by 100, $\gamma_{e,100}$. The modified electron density is then $x_{e,100}' = \gamma_{e,100} x_e$. $p = 2$ is a parameter. $x_{e,100}' \approx 1$ is reasonable according to Falcke et al. [FB95].
- $q_{j/1}$: Total jet power $Q_{\text{jet}}$ in units of the disk luminosity, with a given value of $q_{j/1} = Q_{\text{jet}} / L_{\text{disk}} = 0.15^{+0.2}_{-0.1}$.
- $L_{46}$: Disk luminosity per $10^{46}$ erg/s: $L_{46} = L_{\text{disk}} / (10^{46}$ erg/s).
- $\gamma_{j,5}$: Jet’s Lorentz boost factor, divided by 5. It is assumed to be $(6 \pm 2)/5$.
- $u_3$: Ratio between the total energy density in the jet and the magnetic energy density, divided by a factor three. It is set to $u_3 = 1$ [FMB95].
- $\xi = 0.15$.

These parameters are used to rewrite relation (2.15) as

$$L_{\text{radio}} = \int_0^{3GHz} \, d\nu P_{\text{lobe}} \cdot \nu^{-0.8} = 10^{-26.5 \pm 0.5} \cdot (1 + z)^{3/4} \cdot L_{\text{disk}}^{3/2}. \quad (2.17)$$

assuming a spectral index of $\alpha \approx 0.8$, see section 2.1. A scattering factor of the data has been considered.

Finally, the jet-disk symbiosis can be used to express the generic energy spectrum in terms of the radio luminosity in units of $10^{42}$ erg/s. The result at a luminosity of $L_{42} = 100$ with varying spectral index is shown for steep spectrum sources in figure 2.3 and in figure 2.4 for flat spectrum sources. The power law decrease can be observed up to a cut energy at $E_\nu \sim 10^{11}$ GeV.
2.2. Ingredints of the flux calculation

Figure 2.3: Energy dependence of the generic AGN neutrino spectrum from steep spectrum sources. The normalization is dependent on the particle index $p$, shown for $p = 2.0, 2.2, 2.4, 2.6$. The radio luminosity is assumed to be $L_{42} = 100$.

Figure 2.4: Energy dependence of the generic AGN neutrino spectrum flat spectrum sources. The spectrum is shown at $L_{42} = 100$ and for three different particle indices, $p = 1.0, 1.5, 2.0$. 
2.2.2 AGN RLF for steep spectrum sources

The AGN RLF in the Universe depends on the luminosity and on the redshift. To find the proper relation empirically, a factorizing separation of the density in a luminosity dependent function and a redshift dependent distribution is assumed by Willott et al. [W+00]. The model includes the steep spectrum sources as they are explained in section 2.1. The AGN RLF is given at a frequency of 0.151 GHz. The RLF was assumed to consist of two separate distributions, a low luminosity function including objects without or emission lines and a high luminosity function with objects with strong emission lines. The density $\rho(L_{\text{0,151}}, z)$ is given as the product of a pure luminosity function $\rho_0(L_{\text{0,151}})$ (in units of $1/(\text{Gpc}^3 \Delta \log L_{\text{0,151}})$) and the dimensionless evolution function $f(z)$:

$$\rho(L_{\text{0,151}}, z) = \rho_0(L_{\text{0,151}}) \cdot f(z).$$

Two AGN populations contribute to the RLF:

- **Low luminosity sources** show only weak or no emission lines. These sources are FR-I and weak emission line FR-II galaxies. Above a luminosity $L_{\text{0,151}} = 10^{25.5} \, \text{W}/(\text{Hz sr})$, the sources can almost exclusively be assumed to be FR-II galaxies. For the low luminosity function, $\rho_0(L)$, a an ansatz

$$\rho_0(L_{\text{0,151}}) = \rho_0^0 \left( \frac{L_{\text{0,151}}}{L_\ast} \right)^{-\alpha_l} \cdot \exp \left[ -\frac{L_{\text{0,151}}}{L_\ast} \right]$$

is made. $\alpha_l$ is the power law shape, $\rho_0^0$ is the normalization constant and $L_\ast$ is the break luminosity. The evolution function of the low luminosity population is taken to be

$$f_l(z) = \begin{cases} 
(1 + z)^{k_l} & \text{for } z \leq z_l^0 \\
(1 + z_l^0)^{k_l} & \text{for } z > z_l^0.
\end{cases}$$

The redshift evolution of the sources is known up to $\sim z_l^0$. At higher redshifts, it is assumed to be constant since the actual distribution has not been determined. The are five free parameters used in the functions are listed for two different cosmologies ($\Omega_m = 1$ and $\Omega_m = 0$) in table 2.1. The evolution function is viewed in figure 2.5 while the complete (luminosity and redshift dependent) RLF is shown in figures 2.6 ($\Omega_m = 0$) and 2.7 ($\Omega_m = 1$).

- **The high luminosity population** consists of FR-II galaxies with strong emission lines. The form of the high luminosity function, $\rho_h(L_{\text{0,151}})$, is similar to the one for the low luminosity population. Here, however, the exponential function applies at low luminosities while the power law dominates at higher luminosities:

$$\rho_h(L_{\text{0,151}}) = \rho_h^0 \left( \frac{L - L_{\text{0,151}}}{L_h} \right)^{-\alpha_h} \cdot \exp \left[ -\frac{L_h}{L_{\text{0,151}}} \right].$$

The evolution function of the high luminosity population is assumed to be exponentially increasing up to a certain redshift $z_h^0$ and then continuing as a constant:

$$f_h(z) = \begin{cases} 
\exp \left[ -\frac{1}{2} \left( \frac{z - z_h^0}{z_h^0} \right)^2 \right] & \text{for } z \leq z_h^0 \\
1 & \text{for } z > z_h^0.
\end{cases}$$
The evolution function of the high luminosity population is shown in figure 2.5. It is compared to X-ray data in the following paragraph to justify an exponentially increasing evolution. The complete RLF is viewed in figures 2.6 (\(\Omega_m = 0\)) and 2.7 (\(\Omega_m = 1\)).

The luminosity, which is given here at 0.151 GHz, is used in the integral form for further calculations. Thus the luminosity \(L_{0,151}\) is converted to the integral radio luminosity in units of \(10^{42}\) erg/s, \(L_{42}\), by assuming a spectral index of \(\alpha = 0.8\). Figures 2.6 and 2.7 are given in these units. Furthermore, the RLF is given in units of Gpc\(^{-3}\) (\(\Delta \log(L)\))\(^{-1}\). In following calculations, the RLF will be used per luminosity interval\(^1\).

<table>
<thead>
<tr>
<th>(\Omega_m)</th>
<th>(\rho_l^n)</th>
<th>(\alpha_l)</th>
<th>(L_l^*)</th>
<th>(k_l)</th>
<th>(\rho_h^n)</th>
<th>(\alpha_h)</th>
<th>(L_h^*)</th>
<th>(z_l^n)</th>
<th>(z_h^n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(10^{1.53})</td>
<td>0.542</td>
<td>(10^{26.14})</td>
<td>0.646</td>
<td>4.10</td>
<td>(10^{-6.260})</td>
<td>2.31</td>
<td>(10^{26.95})</td>
<td>1.81</td>
</tr>
<tr>
<td>0</td>
<td>(10^{1.316})</td>
<td>0.581</td>
<td>(10^{26.47})</td>
<td>0.580</td>
<td>3.11</td>
<td>(10^{-6.816})</td>
<td>2.40</td>
<td>(10^{27.36})</td>
<td>1.77</td>
</tr>
</tbody>
</table>

Table 2.1: Parameters used in the calculation of the AGN RLF for steep spectrum sources [W+00]. \(\rho_l^n\) and \(\rho_h^n\) are given in units Gpc\(^{-3}\)\(\Delta \log(L)\). \(L_l^*\) and \(L_h^*\) are given in W/(Hz sr).

Figure 2.5: Evolution function of the low and high luminosity populations versus redshift \(z\). Two cosmologies are shown. The dash-dotted lines represent \((\Omega_m = 1)\) and the solid lines are calculated for \((\Omega_m = 0)\). In both cases, the cosmological constant is \(\Omega_\Lambda = 0\).

\(^1\)\(d \log L = 1/(\ln(10)L) \cdot dL\). Thus \(dn/dL = 1/(\ln(10)L) \cdot dn/(d \log L)\).
Figure 2.6: Radio Luminosity Function versus luminosity for $\Omega_m = 1$ and $z = 0, 1, 2$.

Figure 2.7: Radio Luminosity Function versus luminosity for $\Omega_m = 0$ and $z = 0, 1, 2$. 
2.2. Ingredients of the flux calculation

Testing the Radio Luminosity Function

In this section, the RLF of the steep spectrum sources will be compared to the Radio Luminosity Function calculated from X-ray data. To estimate the number of AGN per comoving volume at a certain redshift \( z \), measurements of AGN up to redshifts of \( z \approx 5.4 \) have been analyzed by Miyaji et al. [MHS00] with the result shown in figure 2.8. In that figure, the analysis results are shown in two different cosmologies, \( \Omega_m = 1 \) and \( \Omega_m = 0.3 \). The data can be fitted by an exponential function with a change in the exponent at \( z = 1.7 \) : At that point, there is a break in the exponential function and the slope changes.

The fits for the two different analyses are

\[
\psi(z, \Omega_m = 1.0) = \begin{cases} 
(11.66 \pm 1.92) e^{(2.81 \pm 0.18) z} \frac{h_{50}}{\text{Mpc}} & \text{for } z < 1.7 \\
(1.34 \pm 0.45) 10^3 e^{(0.05 \pm 0.26) z} \frac{h_{50}}{\text{Mpc}} & \text{for } z > 1.7
\end{cases}
\]  
(2.18)

\[
\psi(z, \Omega_m = 0.3) = \begin{cases} 
(11.63 \pm 1.72) e^{(0.00 \pm 0.14) z} \frac{h_{50}^3}{\text{Mpc}^3} & \text{for } z < 1.7 \\
(1.9 \pm 0.32) 10^3 e^{(0.09 \pm 0.21) z} \frac{h_{50}^3}{\text{Mpc}^3} & \text{for } z > 1.7
\end{cases}
\]  
(2.19)

In this context, \( h_{50} \) is the Hubble parameter in units of 50 km/(s \cdot Mpc). As expected, the deviation from the fit parameters is quite large for higher redshifts (\( z > 1.7 \)).

The data from ROSAT with a matter density of \( \Omega_m = 1 \) can be compared to the high luminosity evolution function of [W+00] (also for \( \Omega_m = 1 \)). Data and evolution function are shown in figure 2.9. Since the evolution function increases exponentially with \( z^2 \), the slope of the evolution function is steeper than the fit for the data points. Although the model does not completely fit the data, the behavior of the two functions (exponential increase up to \( z \approx 1.7 \) and constant function for \( z \lesssim 1.7 \)) is similar which indicates that if the RLF function is used with consistent cosmological parameters the result should not differ significantly.
Figure 2.8: The AGN distribution by [MHS00] is fitted by an exponential function with a bending point at \( z = 1.7 \). Fit parameters are given in equations 2.18 and 2.19.

Figure 2.9: Comparison of data from ROSAT with the evolution function from [W+00]. The exponential increase of both functions can clearly be seen. At high redshifts, \( z > 2 \), the evolution function seems to continue approximately constant.
2.2.3 AGN RLF for flat spectrum sources

171 flat sources from four catalogues at 2.7 GHz are used by Dunlop et al. [DP90] to determine the RLF of flat spectrum blazars. The sample is described in section 2.1. The authors use an \( \{ \Omega_m = 1, \Omega_\Lambda = 0 \} \) cosmology. The ansatz for the flat spectrum RLF is a pure luminosity evolution function of the form

\[
\rho(L, z) = \rho_0 \cdot \left\{ \left( \frac{L}{L_c(z)} \right)^\alpha + \left( \frac{L}{L_c(z)} \right)^\beta \right\}^{-1}.
\]

The function is labeled pure luminosity evolution since only the break luminosity, \( L_c \) evolves with the redshift. Here, \( \alpha \) and \( \beta \) are the power law slopes, \( \rho_0 \) is the normalization and \( L_c(z) \) is the break luminosity which evolves with redshift:

\[
\log[L_c(z)] = a_0 + a_1 \cdot z + a_2 \cdot z^2. 
\]

Here, the luminosity \( L \) is given in units of W/(Hz sr) at a frequency of 2.7 GHz. The RLF is shown in figure 2.10 with a set of parameters given in table 2.2. For further calculations, the luminosity at 2.7 GHz is converted to an integral luminosity by assuming a flat spectrum with \( \alpha_f = 0 \). The RLF is given in units of Gpc\(^{-3} \cdot (\Delta \log(L))^{-1} \), and thus the differential RLF is given as

\[
\frac{dn}{dL}(L, z) = \frac{\rho_0}{\ln(10) \cdot L} \cdot \left\{ \left( \frac{L}{L_c(z)} \right)^\alpha + \left( \frac{L}{L_c(z)} \right)^\beta \right\}^{-1}.
\] (2.20)
Figure 2.10: RLF for flat spectrum sources in a pure luminosity evolution model according to Dunlop et al. [DP90]. Four different redshift values are used, z = 0, 2, 3, 4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_0$</td>
<td>$10^{0.85}$ Gpc$^{-3}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.83</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.96</td>
</tr>
<tr>
<td>$a_0$</td>
<td>24.89</td>
</tr>
<tr>
<td>$a_1$</td>
<td>1.18</td>
</tr>
<tr>
<td>$a_2$</td>
<td>-0.28</td>
</tr>
</tbody>
</table>

Table 2.2: Parameters for the flat spectrum RLF [Pea85].
2.2.4 Cosmological tools

This section will give a brief overview of the cosmological tools used in the calculation of the neutrino flux. The cosmological parameters which will be used in the following are described in chapter.

The comoving volume per redshift, $dV_c/dz$ in a $\Lambda \neq 0$ and $\Omega_k = 0$ cosmology is given by [CPL92]

$$
\frac{dV_c}{dz} = \frac{4\pi}{(1+z)^2} \frac{c^2}{H_0} \left[ (1+z)^2 \cdot (1 + \Omega_m \cdot z) - \Omega_\Lambda \cdot z \cdot (2 + z) \right]^{-1/2}.
$$

The luminosity distance, $d_L$, can be expressed in terms of redshift as follows [CPL92]:

$$
d_L = \frac{(1+z) \cdot c}{H_0} \cdot I(z)
$$

with

$$
I(z) := \int_0^z [(1 + z')^2 \cdot (1 + \Omega_m z') - \Omega_\Lambda z' \cdot (2 + z')]^{-1/2} dz'.
$$

Thus the product of $dV_c/dz \cdot 1/(4\pi d_L^2)$ is given as

$$
\frac{dV_c/dz}{4\pi d_L^2} = \frac{1}{(1+z)^2} \cdot \frac{c}{H_0} \left[ (1+z)^2 \cdot (1 + \Omega_m \cdot z) - \Omega_\Lambda \cdot z \cdot (2 + z) \right]^{-1/2}.
$$

It has to be noted that the factor $1/(4\pi d_L^2)$ is given in units of $1/(\text{sr} \cdot \text{cm}^2)$ while $dV_c/dz$ is given in units of $\text{Gpc}^3$. Provided that $c/H_0 \approx 3 \text{ Gpc}/\text{h}$ (with $H_0 = 100 \cdot h \cdot \text{km}/(\text{s \cdot Mpc})$ and $c \approx 3 \cdot 10^8 \text{ km/s}$) and converting Gigaparsec into centimeters in the luminosity distance equation (2.23) can be transformed into

$$
\frac{dV_c/dz}{4\pi d_L^2} = \frac{3.15 \cdot 10^{-55}}{h \cdot (1+z)^2} \cdot \left[ (1+z)^2 \cdot (1 + \Omega_m \cdot z) - \Omega_\Lambda \cdot z \cdot (2 + z) \right]^{-1/2} \left[ \frac{\text{Gpc}^3}{\text{cm}^2 \cdot \text{sr}} \right].
$$

Furthermore, the redshift evolution of the neutrino energy due to the expansion of the Universe has to be considered:

$$
E_\nu = E_\nu^0 \cdot (1 + z).
$$

$E_\nu^0$ is the energy of the particle when it arrives at the detector.

2.2.5 Integration limits

The differential flux given in equation (2.5) is integrated over luminosity and redshift. The integration over the luminosity has to be performed before the redshift integration, since the lower luminosity limit implicitly depends on the redshift. The limits for the $z$-integration are taken to be

$$
z_{\text{min}} = 0.03, \quad z_{\text{max}} = 6.
$$

The minimum of the $z$-integration is given by the fact that the flux is assumed to be diffuse. For $z < 0.03$, a non-diffuse contribution from the supergalactic plane is
<table>
<thead>
<tr>
<th></th>
<th>$C$ [GeV]</th>
<th>$\beta$</th>
<th>$E_{\nu}^{\text{cut}}$ [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>steep</td>
<td>$1 \cdot 10^4 \cdot (1 + z)^{-1/4}$</td>
<td>$1/3$</td>
<td>$3.2 \cdot 10^{10}$</td>
</tr>
<tr>
<td>flat</td>
<td>$7.5 \cdot 10^9$</td>
<td>0.3935</td>
<td>$1.3 \cdot 10^9$</td>
</tr>
</tbody>
</table>

Table 2.3: Parameters for the determination of the lower luminosity integration limit.

expected. The maximum redshift is taken as $z = 6$, since the total $z$ dependence decreases rapidly with redshift and any contribution above $z_{\text{max}}$ can be neglected as will be shown in the following. Up to $z = 6$, the consistency of the used models is ensured, since AGN have been detected up to redshifts of $z \approx 6.4$ [WMJ03].

The maximum radio luminosity limit is taken to be $L_{42}^{\text{max}} = 10^4$. The contribution for $L_{42} > 10^4$ is negligible as it will be shown in section 2.4 and for all following calculations, $L_{42}^{\text{max}} = 10^4$ is used. The lower radio luminosity limit depends on the following facts:

1. The absolute lower limit: For the model of extended sources, the jet-disk symbiosis is developed for FR-II galaxies and the absolute lower limit, $L_{42, \text{min}}^{f}$, is given by the lower luminosity limit for FR-II galaxies which is given as $L_{0.178} = 2.5 \cdot 10^{26}$ W/Hz with an integral radio luminosity of approximately $L_{42, \text{min}}^{f} = 10$. The sample of flat spectrum sources, on the other hand, ranges from $10^{-1.5} < L_{42} < 10^4$ and the absolute lower limit, $L_{42, \text{min}}^{f}$, is thus $L_{42, \text{min}}^{f} = 10^{-1.5}$.

2. The maximum proton energy which can be produced by an AGN is connected with the radio luminosity according to Lovelace et al. [Lov76]:

$$E_{p}^{\text{max}} = C' \cdot \sqrt{L_{\text{disk}}}.$$  

(2.25)

This result is supported by the jet-disk symbiosis model. Taking the highest observed energy so far, $E_{p} \approx 10^{21}$ eV and the maximum luminosity, $L_{\text{disk}}^{\text{max}} = 10^{47}$ erg/s, the constant $C'$ is determined to be $C' = 10^{11.5}$ GeV/\sqrt{erg/s}. When the jet-disk symbiosis model by Falcke et al. is used and the ratio between $E_{p}$ and $E_{\nu}$ taken to be 20/1, the maximum neutrino energy is correlated to the radio luminosity as

$$E_{\nu}^{\text{max}} = C \cdot L_{42}^{\beta}.$$  

The lower luminosity limit of an AGN contributing to a certain flux at a certain neutrino Energy is

$$L_{42}^{\text{min}} = \max\{ L_{42, \text{min}}^{f}, \left( \frac{E_{\nu}}{C} \right)^{1/\beta} \},$$  

(2.26)

Since the RLF is decreasing with luminosity, the energy spectrum will steepen from that energy on, when the luminosity-energy relation exceeds the value of the absolute luminosity limit, $(E_{\nu}/C)^{1/\beta} \geq L_{42, \text{min}}^{f}$. This is the case for an energy

$$E_{\nu}^{\text{cut}} \approx (L_{42, \text{min}}^{f})^{\beta} \cdot C \text{ GeV}$$  

(2.27)

in the comoving frame. The cut energies for the two models are given in table 2.3.
2.3 Constraints and limits on the extragalactic neutrino flux

In this section, current experimental limits will be discussed.

Figure 2.11: Measurement of the atmospheric neutrino flux up to $\sim 10^5$ GeV [Gee03]. The data fit the predicted atmospheric neutrino flux - the upper curve gives the horizontal flux while the lower line is the vertical flux [VZ80, H+95]. The three horizontal limits are calculated assuming an $E^{-2}$ neutrino spectrum from extragalactic sources.

- **AMANDA**
  Current limits on the extragalactic neutrino flux are given by the AMANDA II experiment [RA03] as is shown in figure 2.11. A diffuse limit on the extragalactic muon neutrino flux could be derived in the range of 6 TeV - 1 PeV, assuming an $E^{-2}$ spectrum:

  \[ E^2\Phi(E_\nu) = 8 \cdot 10^{-7} \frac{GeV}{cm \cdot s \cdot sr}. \]

  Between 80 TeV and 7 PeV, a limit of

  \[ E^2\Phi(E_\nu) = 9 \cdot 10^{-7} \frac{GeV}{cm \cdot s \cdot sr}. \]

can be set for the sum of all flavor neutrinos producing huge electromagnetic or hadronic cascades.
For energies around 1 PeV to 10 PeV, the Earth becomes opaque to neutrinos, since the neutrino-nucleon cross section rises with increasing neutrino energy. The neutrino interacts with a nucleon on its way through Earth. From the search of bright multi-muon events from near the horizon or from the upward hemisphere, a limit of

$$E^2 \Phi(E_\nu) = 7.2 \cdot 10^{-7} \frac{GeV}{cm \cdot s \cdot sr}$$

in an energy range of 2.5 PeV–5.6 EeV can be set for muon neutrinos.

The sensitivity of AMANDA-II for muon neutrinos for one year (2000) is expected to be

$$E^2 \Phi(E_\nu) = 2 \cdot 10^{-7} \frac{GeV}{cm \cdot s \cdot sr}.$$

- **EGRET**

According to EGRET measurements \([L^+99b]\), the isotropic diffuse gamma ray background above 30 MeV of an energy \(E_\gamma\) is given as

$$E^2 \frac{dN}{dE_\gamma} = (1.37 \pm 0.06) \cdot 10^{-6} \cdot E_\gamma^{-0.1 \pm 0.03} GeV \ cm^{-2} s^{-1} sr^{-1}.$$

To estimate the maximum neutrino flux resulting from sources with the given gamma ray spectrum, the integral fluxes are compared assuming photoproduction of pions on a power law target with \(\alpha = 1\) \([LM00]\):

$$\int_{30 \ MeV} dN \frac{dE_\gamma}{dE_\nu} \sim 2 \cdot \int \frac{dN}{dE_\nu} \ dE_\nu.$$

### 2.4 Results from the neutrino flux calculation

The integral, isotropic neutrino flux is given in equation (2.5). In the following section, the resulting neutrino spectrum will be discussed for steep and flat spectrum sources.

#### 2.4.1 Dependence on the parameters

In the following section, the parameters of the calculation will be discussed. The integration limits will be varied at a constant energy of \(E^0_\nu = 10\) GeV. The redshift is set to \(z = 2\) in all calculations taking into account the luminosity limits. In computations of the redshift limit dependences, the luminosity will be set to \(L_{42} = 100\).

- **Steep spectrum sources**

  The integration over the luminosity is done numerically. The result depending on the upper integration limit is shown in figure 2.12. At \(L_{42}^{max} > 10^3\), the result is independent of the upper integration limit. The dependence of the flux on the lower integration limit is shown in figure 2.13. As the lower integration, \(L_{42}^{min} = 10\) is chosen since only FR-II galaxies are considered. The differential flux, \(d\Phi/dz\) is presented in figure 2.14. The maximum contribution to the integral flux results from redshifts between \(z = 0\) and \(z = 2\). Higher redshifts do not contribute as strongly. This is also indicated by the dependence of the flux on the upper redshift limit \(z_{max}\), shown in figure 2.15.
2.4. Results from the neutrino flux calculation

Figure 2.12: Dependence of the steep spectrum sources on the upper luminosity limit at a redshift of $z = 2$. The local energy is chosen to be $E^0_\nu = 10$ GeV. Four different particle indices, $p = 2.0, 2.2, 2.4, 2.6$ are discussed. The flux is independent of the upper integration limit for $\log(L_{\text{max}}) < 3$.

Figure 2.13: Variation of the steep spectrum on the lower luminosity limit at a redshift of $z = 2$ and at a local energy $E^0_\nu = 10$ GeV. Again, four the four particle indices, $p = 2.0, 2.2, 2.4$ and 2.6 are used.
Figure 2.14: Differential flux $d\Phi/dz$ of the steep spectrum sources versus $z$ for different indices $p$. The energy was set to $E_\nu^0 = 10$ GeV.

Figure 2.15: Neutrino flux with variation of the upper redshift integration limit $z_{\text{max}}$ for steep spectrum sources. The neutrino energy is set to $E_\nu^0 = 10$ GeV while the particle index is varied as $p = 2.0, 2.2, 2.4, 2.6$. 
2.4. Results from the neutrino flux calculation

- Flat spectrum sources

The luminosity dependence of the flat spectrum source sample is shown in figures 2.16 and 2.17. In figure 2.16, the maximum luminosity is varied at a constant lower limit \( L_{\text{min}} = 10^{-1.5} \). The flux increases up to a maximum luminosity of \( L_{42}^{\text{max}} \sim 10^2 \). Sources at higher redshifts do not contribute. In figure 2.17, the lower integration limit is varied at a constant upper limit \( L_{\text{max}} = 10^4 \). The total \( z \)-dependence of the flat spectrum flux is shown in figure 2.18. Near AGN sources contribute more than those sources which are farther away. The integral spectrum versus the upper redshift integration limit is shown in figure 2.19. The flux rises approximately as a power law up to \( z_{\text{max}} = 1 \). Sources at higher redshift do not contribute significantly.

The differential spectra and the dependence of the flux on the upper redshift integration limit show that a contribution to the neutrino flux above \( z \sim 6 \) is negligible for both types of source spectra.

\[
\frac{d\Phi}{dz} \times 10^{-8}
\]

\[\begin{align*}
\text{p} = 1.5 \\
\text{z} = 2 \\
E_{\nu} = 10 \text{ GeV}
\end{align*}\]

Figure 2.16: Neutrino flux with variation of the maximum luminosity of the flat spectrum flux. The flux is evaluated at an energy of \( E_{\nu} = 10 \text{ GeV} \) and at a redshift of \( z = 2 \). The result is independent of the upper limit for \( \log(L_{42}^{\text{max}}) \gtrsim 1.5 \).
Figure 2.17: Dependence of the flat spectrum AGN flux on the lower integration limit at a redshift of $z = 2$ and a local energy of $E^0_p = 10$ GeV. The particle index is taken to be $p = 1.5$.

Figure 2.18: Differential flat spectrum AGN flux $d\Phi/dz$. A spectral index of $p = 1.5$ is used and setting the local energy to $E^0_p = 10$ GeV.
2.4. Results from the neutrino flux calculation

Figure 2.19: Dependence of the flat spectrum source flux on the upper redshift integration limit $z_{\text{max}}$. The spectral index is $p = 1.5$ and the local energy is chosen to be $E_\nu^0 = 10$ GeV.
2.4.2 The final spectra

![Graph showing neutrino spectrum from steep spectrum AGN. The particle index is varied (p = 2.0, 2.2, 2.4, 2.6).]

Figure 2.20: Neutrino spectrum from steep spectrum AGN. The particle index is varied (p = 2.0, 2.2, 2.4, 2.6).

The final steep spectrum source flux is shown in figure 2.20. The spectrum is presented for four different particle indices, p = 2.0, 2.2, 2.4, 2.6. Here, p = 2 is a commonly used value in literature and p = 2.6 has been computed in section 2.1 from the spectral index at 5 GHz for the steep spectrum population. The spectrum decreases as a power law up to a cut energy at approximately $10^{10}$ GeV. At higher energies, the spectrum is vanishing exponentially. For a particle index of $p = 2$, the flux is approximately

$$E^2 \Phi \approx 10^{-7} \text{GeV}/(\text{s sr cm}^2).$$

This is about an order of magnitude lower than current neutrino flux limits.

The flat source neutrino spectrum is shown in figure 2.21 with a spectral index of $p = 1.5$. Again, the spectrum decreases with its index up to the maximum energy at $\sim 10^{10}$ GeV and vanishes exponentially at higher energies.
2.4. Results from the neutrino flux calculation

The muon neutrino spectrum including flat and steep spectrum sources is shown in figure 2.22. One third of the contribution to the total neutrino flux comes from the muon neutrinos. The spectrum is dominated by steep spectrum sources up to energies of $10^6$ GeV, represented as the dash-dotted line. This part is clearly surpressed by the atmospheric neutrino flux. At higher energies, most of the flux arises from flat spectrum sources, presented by the dotted line. The limits and the measured spectrum is also indicated. The flux lies several orders of magnitude below the limits. An observation of the neutrino flux at energies close to the measured spectrum would require improved reconstruction. At higher energies $E \sim 10^6$ GeV $- 10^9$ GeV, the spectrum could be observed by experiments such as IceCube which are sensitive to such high energies due to the large volume of 1 km$^3$. Figure 2.23 shows the AGN neutrino flux model with a constant particle index of $p = 2$. Here, the spectrum is dominated by steep spectrum sources at all energies. In that case, current neutrino experiments should be able to observe the AGN flux component with sensitivities of an order of magnitude higher than for current measurements.

Figure 2.21: Neutrino spectrum from flat spectrum AGN. The particle index is taken to be $p = 1.5$. 

\[ L_{\nu} = 10^{-1.5} \]
Figure 2.22: AGN neutrino spectrum. The dash-dotted line represents the steep spectrum sources, $p = 2.6$. The dotted line shows the flat spectrum sources with $p = 1.5$. Data are given by the AMANDA Experiment (2000). The dashed line represents the conventional atmospheric spectrum. Horizontal lines are AMANDA limits as explained in figure 2.11, section 2.3.
Figure 2.23: AGN neutrino spectrum with an index of $p = 2$. The flat spectrum sources (dotted line) do not contribute significantly compared to the steep spectrum population (solid line).
Chapter 3

Uncertainties in Neutrino Flux Calculations

Recent observations of the CMB multipole spectrum have significantly constrained the cosmological parameters (see section 1.4). The topic of this section will be to examine the dependence of neutrino flux calculations on the choice of cosmological parameters. In this chapter, the redshift depending functions are considered without taking into account the redshift evolution of the neutrino energy. The Hubble parameter will be assumed to be $h = 0.5$. In the first part of the chapter, the AGN evolution function by Miyaji will be applied [MHS00]. In the second part, both redshift and luminosity dependence of the radio luminosity function will be used in the RLF model of Willott et al. [W⁺00]. It will be shown that with the consistent choice of the radio luminosity function and cosmological parameters, the neutrino flux will be independent of the choice of cosmological parameters. Three different cosmologies will be used:

- **Einstein-de Sitter (EdS)**
  The former standard cosmology is based on the Einstein-de Sitter universe with
  \[
  (\Omega_m, \Omega_\Lambda) = (1, 0).
  \]
  Most AGN neutrino flux calculations are based on this cosmology (e.g. [MPR01, Man95, RB93]).

- **Empty universe ($\Omega_m = 0$)**
  \[
  (\Omega_m, \Omega_\Lambda) = (0, 0, 0)
  \]
  is chosen to see if there is much deviation from a $\Omega_\Lambda \neq 0$ cosmology.

- **$\Omega_m = 0.3$ cosmology**
  \[
  (\Omega_m, \Omega_\Lambda) = (0.3, 0.0)
  \]

In all cases, the Hubble parameter is taken to be $h = 0.5$. 

Ende: Jim Knopf und die Wilde 13
3.1 Redshift dependence of AGN neutrino flux calculations using Miyaji’s evolution function

The AGN distance distribution by Miyaji et al. is given in the EdS and in the $\Omega_m = 0.3$ cosmology. These two approaches will be compared using Miyaji’s model [MHS00]. The AGN evolution function given in section 2.2.2 will be used in this section to examine uncertainties in neutrino flux calculations due to cosmological parameters. Two cosmologies are available from Miyaji et al., the Einstein-de Sitter cosmology and the $\Omega_m = 0.3$ cosmology.

Neglecting the redshift evolution of the neutrino energy, the (absolute) total $z$-dependence is shown in figure 3.1. The function peaks at the cut redshift $z = 1.7$ of the evolution function. The ratio of the two cosmologies is presented in figure 3.2. It increases with the redshift.

Figure 3.1: Approximate redshift dependence of the neutrino flux in the EdS and $\Omega_m = 0.3$ cosmology. The function peaks at $z = 1.7$.

Keeping the upper integration limit variable, the redshift dependence of the neutrino flux can be integrated over redshift with the result shown in figure 3.3. The function increases rapidly up to $z_{\text{max}} \sim 2$ and flattens at higher redshifts. The ratio between both models is shown in figure 3.4. The models differ less than a factor of three and the difference increases with redshift. At a maximum redshift of $z_{\text{max}} = 6$, the ratio of the two models is $\text{ratio} \sim 2.8$. 
Figure 3.2: Ratio of the z-dependences of the two models, $\Omega_m = 0.3$ and EdS. The ratio increases with redshift.

Figure 3.3: Integration over approximate redshift dependence of the neutrino flux in the two cosmologies, EdS and $\Omega_m = 0.3$. 
3.2 Neutrino flux using Willott’s RLF

The Einstein-de Sitter cosmology will be compared with the empty Universe cosmology using Willott’s model [W+00]. The Radio Luminosity Function by Willott will be used to show that neutrino flux calculations are basically independent on the choice of cosmology, if the same cosmology is used throughout the whole calculation. The flux calculations will be done by using a simple $E^{-2}$ neutrino spectrum. The energy dependence of the spectrum does not influence the relative difference of neutrino flux models in different cosmologies. The flux per redshift at 10 GeV for the two given cosmologies, $\Omega_m = 0$ and EdS, is shown in figure 3.5. The two models differ most significantly at high redshifts. The integrated spectrum is shown in figure 3.6. Since the dominant contribution comes from sources at $z < 2$, the integrated spectra differ less than a factor of two.

3.3 Conclusions

Both the calculation from section 3.1 and 3.2 give indications that the choice of cosmological parameters does not influence the normalization of a neutrino flux model more than a factor of $2 - 3$ does. Since AGN radio luminosity functions are very hard to determine due to uncertainties in the determination of the sources redshift and absolute luminosities, a factor of $2 - 3$ lies certainly within the range of these errors.
Figure 3.5: Redshift dependence of the neutrino spectrum at 10 GeV in two cosmologies, EdS and $\Omega_m = 0$. The solid line shows $\Omega_m = 0$ and the dashed line represents an Einstein-de Sitter Universe. The two spectra deviate most at higher redshift. At these redshifts, however, the contribution to the total flux becomes negligible.

Figure 3.6: Neutrino spectrum in two different cosmologies. The solid line represents the $\Omega_m = 0$ cosmology, the dashed line shows the model in an EdS Universe.
3.3. Conclusions
Chapter 4

Large Volume Neutrino Experiments

In this chapter, current and future large volume neutrino experiments will be presented. All detectors use large arrays of dense matter (water, ice, salt) to observe muons which are produced when neutrinos interact with protons or neutrons. Also, neutrino induced electromagnetic cascades can be observed at higher energies.

Since the cross section for such an interaction is very small as well as the event rate at high energies, large arrays are needed to get a significant neutrino rate. Current experiments measure the atmospheric neutrino spectrum while these and following experiments hopefully are able to detect the extraterrestrial neutrino flux component in the future. The main properties of current and future experiments are listed in table 4.1 and 4.2. Short forms are explained below.

<table>
<thead>
<tr>
<th>Detector</th>
<th>depth [m]</th>
<th>$E_{\mu}^{\text{max}}$ [GeV]</th>
<th>Volume [km$^3$]</th>
<th>Neutrinos</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMANDA-II</td>
<td>1500 - 2000</td>
<td>50</td>
<td>0.07</td>
<td>$\nu_\mu$, $\nu_e$, $\nu_\tau$</td>
</tr>
<tr>
<td>ANTARES</td>
<td>2050 - 2400</td>
<td>10</td>
<td>0.03</td>
<td>$\nu_\mu$, $\nu_e$, $\nu_\tau$</td>
</tr>
<tr>
<td>Baikal</td>
<td>1070</td>
<td>15</td>
<td>0.0001</td>
<td>$\nu_\mu$</td>
</tr>
<tr>
<td>IceCube</td>
<td>1400 - 2400</td>
<td>500 - 1000</td>
<td>1</td>
<td>$\nu_\mu$, $\nu_e$, $\nu_\tau$</td>
</tr>
<tr>
<td>NESTOR</td>
<td>4000</td>
<td>4</td>
<td>0.008</td>
<td>$\nu_\mu$, $\nu_e$, $\nu_\tau$</td>
</tr>
<tr>
<td>RICE</td>
<td>100 - 300</td>
<td>$10^6$</td>
<td>0.008</td>
<td>$\nu_\mu$</td>
</tr>
<tr>
<td>SALSA</td>
<td></td>
<td></td>
<td></td>
<td>$\nu_e$, $\nu_\tau$</td>
</tr>
</tbody>
</table>

Table 4.1: Properties of UHE neutrino experiments (1).
4.1. Optical Cherenkov Light Detection

<table>
<thead>
<tr>
<th>Detector</th>
<th>Location</th>
<th>Status</th>
<th>Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMANDA-II</td>
<td>S. Pole</td>
<td>RT</td>
<td>OCI</td>
</tr>
<tr>
<td>ANTARES</td>
<td>France</td>
<td>T</td>
<td>OCW</td>
</tr>
<tr>
<td>Baikal</td>
<td>Siberia</td>
<td>RT</td>
<td>OCW</td>
</tr>
<tr>
<td>IceCube</td>
<td>S. Pole</td>
<td>P</td>
<td>OCI</td>
</tr>
<tr>
<td>NESTOR</td>
<td>Greece</td>
<td>T</td>
<td>OCW</td>
</tr>
<tr>
<td>RICE</td>
<td>S. Pole</td>
<td>T</td>
<td>RCI</td>
</tr>
<tr>
<td>SALSA</td>
<td>probably Texas</td>
<td>UD</td>
<td>RCS</td>
</tr>
</tbody>
</table>

Table 4.2: Properties of UHE neutrino experiments (II).

SHORT FORMS

OCI optical Cherenkov radiation in ice
OCW optical Cherenkov radiation in water
P Proposed
RCS radio Cherenkov radiation in salt
RCW radio Cherenkov radiation in water
RT Running and taking data
T Testing and development
UD Under discussion

4.1 Optical Cherenkov Light Detection

Figure 4.1: The topology of a muon track (left) differs from a cascade event (right). The muon carries a Cherenkov cone with an angle of $\theta_C \approx 41^\circ$, while cascade events show a spherical pattern in the detector [Kow04].
Muon neutrinos traveling through water or ice can interact with the medium and produce muons. In a water-like medium, the muon travels faster than light in the same medium and is therefore producing a Cherenkov cone. The Cherenkov light emitted by the muon can be detected by an array of Optical Modules (OMs) consisting of Photo Multiplier Tubes (PMTs) embedded in pressure resistant glass shells. The spacing of the PMTs is determined by the absorption length of the Cherenkov light in the medium. The size of the array is chosen to be as large as possible since the neutrino flux decreases with energy. Further, the detection rate of (ultra) high energy events increases with the detector size. Another problem is how to distinguish between neutrino induced muons and atmospheric muons from CR interactions. This is solved by using the Earth as a filter: Down going events have a very large rate of CR muons\(^1\) while up-going events can be considered to be neutrino induced muons since all CR muons will decay while transversing the Earth. Thus, using large natural water or ice volumes is a very efficient method to measure the neutrino flux.

At high energies, \(E_\nu \sim EeV\), the Earth becomes opaque for neutrinos (see chapter 5). However, events from the horizon \((\theta = 90^\circ)\) can still be observed.

Electron and tau neutrinos can also be observed in large under water neutrino detectors: When these neutrinos interact with matter, secondary leptons are produced, inducing an electromagnetic cascade. The cascade can be seen indirectly by the emitted Cherenkov light of the cascade particles. Cascade events have a sphere like evolution, while muon neutrinos going through the detector appear as a Cherenkov cone (see figure 4.1). Since the extension of a cascade like event is small, the cascade vertex has to lie inside of the detector to get an identifiable signal. The principle of a neutrino detector in a water-like medium can be explained with the example of the AMANDA (Antarctic Muon and Neutrino Detector Array) experiment which is located near the geographical South Pole at approximately 1500 – 2000 m depth in the ice [CA02]. This depth is primarily necessary in order to reduce the background rate of down-going muons and secondly, the ice can be considered to be sufficiently without bubbles or other disturbing features and the Cherenkov light travels in straight lines without being refracted. The AMANDA-A detector was built at a depth of approximately 600 – 800 m as a prototype detector. The ice is not bubble-less at this depth and the successors were placed deeper in the ice. The AMANDA-B10 detector completed in January 1997 consists of 302 optical modules (OMs) attached on ten strings. The strings are arranged cylindrically with a diameter of 120 m and a height of 500 m. The AMANDA-B10 array was enlarged in January 2000 to the AMANDA-II detector with 677 OM on 19 strings. A model of the AMANDA detector is shown in figure 4.2. The effective area of a detector is a measure for its efficiency. In chapter 5, it will be used to determine the rate of detectable neutrinos. The effective area for muon neutrinos, \(A_{\text{eff}}\), can be calculated in dependence of the muon energy \(E_\mu\). It is defined as the generic area weighted with the ratio of the number of detected events and the number of generated events.

The future project, IceCube is an enlargement of the AMANDA detector to a 1 km\(^3\) neutrino detector with 4800 OM on 80 strings. A volume of 1 km\(^3\) is expected to be sufficiently sensitive for the detection of the extraterrestrial neutrino flux, especially to the extragalactic component such as the AGN or GRB flux. Predictions about the

\(^1\)The ratio between raw triggered muons and neutrino induced muons in AMANDA is \(10^6 : 500!\)
neutrino event rate from AGN or other extragalactic objects (GRB, TD, etc.) are therefore quite important.

Other experiments are being built in water, such as Baikal, NESTOR and ANTARES [DB02, BN98, Mon03]. The Baikal experiment is located in the Baikal lake, 3.6 km away from the shore at 1.1 km depth [DB02]. The detector has an effective area of approximately $\sim 10^3 - 5 \cdot 10^3$ m$^2$ and has an umbrella like frame. On each of the eight strings 24 pairs of PMTs are mounted. Two PMTs in a pair are switched in coincidence to suppress the bioluminescence and the PMT noise. The OMs in the Baikal experiment have to lie close to each other since the absorption length of Cherenkov light in lake water is only 8 m. The advantage of WIMP search is given due to the low energy threshold.

The ANTARES project (Astronomy with a Neutrino Telescope and Abyss environmental RESearch) started in 1996 and will be located at a depth of 2400 m in the Mediterranean Sea, 37 km off-shore of La Seyne sur Mer which is near Toulon (France). It has a surface area of 0.1 km$^2$ and is supposed to be completed by the end of 2004 [Mon03]. Once complete, ANTARES will consist of ten strings with totally 1000 PMTs.

NESTOR (Neutrino Extended Submarine Telescope with Oceanographic Research) is also an underwater neutrino detector which is located in the Ionian Sea south west of Peloponnesos (Greece) [BN98]. The detector is planned to be build in 3800 m depth, 13 nautical miles from the coast. The first phase of the detector consists of a hexagonal tower with 168 OMs. The second step foresees the deployment of six more towers around the first one with a distance of 140 m between the towers and a total of 1176 PMTs. The final goal is to reach an array of 13 towers and 24 strings with totally 2760 OMs and a surface of 1 km$^2$. One tower consists of twelve hexagonal semirigid floors with a spacing of 30 m in between the floors. The PMTs are paired, one PMT looks up and the other down. This leads to a total up-down symmetry.

4.2 Radio Cherenkov Radiation

A theory of Cherenkov radiation at radio frequencies was developed in the 1960s by G. A. Askaryan [Ask62]. An electromagnetic bunch is caused by a neutrino and knocks out electrons from the molecules in the atmosphere due to Compton scattering. Positrons from pair production annihilate with electrons in the atoms which leads to a negative charge excess of approximately 30%. A bunch of these charged particles has a diameter of $\sim 10$ cm [Sal]. Looking at the power $P$ of Cherenkov radiation, it is proportional to frequency $\nu$ and bandwidth $\Delta \nu$:

$$ P \propto \nu \cdot \Delta \nu. $$

The refraction index for light in a medium changes back to $n \approx 1$ at frequencies above the UV light and the differential power decreases. Therefore, Cherenkov radiation is optically seen as blue light. Since the power is proportional to the squared electrical

\[2\text{In comparison, the absorption length in the antarctic ice is 100 m.}\]

\[3\text{Weakly Interacting Massive Particles (WIMPs) are assumed to contribute to dark matter. These particles are very massive and they assemble in gravity centers such as center of the Earth or the sun. Heavy supersymmetric particles are WIMP candidates for instance.}\]
field, 

\[ P \sim |\mathbf{E}|^2 \]

and for optical frequencies, \( |\mathbf{E}| \sim \sqrt{N} \) with \( N \) as the number of particles, the power is proportional to \( N \):

\[ P_{\text{opt}} \propto N. \]

For lower frequencies the corresponding wavelength is bigger than the size of the bunch \((\lambda \geq d)\) and the electromagnetic fields of the particles are in phase \([S+01]\) and it follows \( |\mathbf{E}| \propto N \) and

\[ P_{\text{rad}} \propto |\mathbf{E}|^2 \propto N^2. \quad (4.1) \]

Therefore, the ratio between radio power \( P_{\text{rad}} \) and optical power \( P_{\text{opt}} \) of the Cherenkov light is

\[ \frac{P_{\text{rad}}}{P_{\text{opt}}} = N \left( \frac{\nu_{\text{rad}}}{\nu_{\text{opt}}} \right)^2. \quad (4.2) \]

Typical frequencies are \( \nu_{\text{rad}} = (100 - 500) \) MHz (corresponding to wavelength \( \lambda_{\text{rad}} = (3 - 0.6) \) m) and \( \nu_{\text{opt}} = 75 \cdot 10^9 \) MHz (optical blue light, \( \lambda_{\text{opt}} = 400 \) nm). Thus, the ratio between optical and radio power is

\[ \frac{P_{\text{rad}}}{P_{\text{opt}}} \approx (1.8 - 44.4) \cdot 10^{-20} N. \quad (4.3) \]

Although a particle excess of \( N \approx 10^{18} - 10^{20} \) would be necessary to reach an intensity equivalent to the optical intensity, which would imply an incident particle energy of \( \sim 10^{26} \) eV, recent experiments show that such high intensities are not needed for a detectable Cherenkov signal \([S+01]\). Therefore, the detection of the radio signal of Cherenkov radiation is an alternative method which is already used RICE \((\text{Radio Ice Cherenkov Experiment})\), located in the Antarctic ice. The RICE experiment was initiated in 1995 by the AMANDA collaboration which consented to co-deployment of two dipole radio receivers in the first holes for the AMANDA strings \([K+03]\). Today, the RICE experiment consists of a 16-channel array of dipole receivers within a volume of \((200 \text{ m})^3 \) \([K+03]\).

Another approach is using large salt mines as a natural detector for Cherenkov light. Optical Cherenkov radiation is obviously absorbed while the radio signal would be detectable by helical radio antennas \([B+02]\). Salt has a density of \( \rho_{\text{salt}} = 2.2 \rho_{\text{water}} \) where \( \rho_{\text{water}} \) is the water density and thus, the interaction probability in a 1 km\(^3\) salt detector is higher than for a 1 km\(^3\) water/ice detector. Furthermore, there are tests going on in US salt mines about the properties of the salt which show that the radio Cherenkov radiation does not bend significantly on salt layers or impurities. Although a proposal for a certain experiment has not been made yet, a project - in literature referred to as SALSA \((\text{SALtdome Shower Array})\) - is thought through. The basic concept is to take 10 by 10 strings with 10 antenna nodes per string and a spacing of approximately 250 m.
Figure 4.2: Schematic drawing of the AMANDA detector. [CA02].
Chapter 5

Expected Event Rates for Neutrino Telescopes

Obviously, not all neutrinos arriving at the Earth’s surface interact and can be observed by a detector. The event rate depends on the incident spectrum and the neutrino cross sections. The aim of this chapter is the calculation of the neutrino event rate for various large volume neutrino detectors. In order to do that, the neutrino induced lepton flux through the detector has to be determined. The lepton spectrum is a convolution of the neutrino flux with the Earth shadow factor and the probability for the neutrino to produce a lepton in the vicinity of the detector. These three functions will be discussed considering interactions of muon and electron neutrinos with nucleons. Tau neutrinos are observable as well, but these events are not included in the following discussion. In case of anti-electron neutrinos, the interaction with electrons via resonant scattering at \( \sim 6.3 \text{ PeV} \) has to be considered. Finally, the event rate will be determined for various detector arrays.

An overview of neutrino-nucleon scattering is given in appendix B.2. Further, the results of the calculations made in this section are summarized there.

5.1 Weak interactions

The dominant contribution to the high energy neutrino cross section in matter is deep inelastic scattering with hadrons. In this section, the differential and total cross section for this process will be described within the parton model [PS95, Rob90]. The corresponding Feynman graph is shown in figure 5.1.

Some variables are defined below since they will be used frequently in the following calculations.

- The energy loss of the incident neutrino in the lab frame is

  \[ \nu = \frac{q \cdot p}{m_N} = E_{\nu} - E_l. \]

  Here, \( m_N \) and \( p \) are mass and four momentum of the interacting nucleon while
5.1. Weak interactions

![Diagram of weak interaction](image)

Figure 5.1: A neutrino of momentum $k$ interacts with a nucleon of momentum $p$, resulting in a lepton of momentum $k'$ and several hadrons.

$E_\nu$ and $E_l$ are the neutrino’s and the lepton’s energy. $q$ is the momentum transfer four vector.

- If $q^2$ is the momentum transfer, the positive variable

$$Q^2 = -q^2 = 2(E_\nu E_l - \mathbf{k} \cdot \mathbf{k'} - m_\nu^2 - m_l^2)$$

can be defined. $k$ and $k'$ are the neutrino’s and the lepton’s four momentum; $m_\nu$ and $m_l$ are the corresponding masses. Since the neutrino mass is $m_\nu \approx 0$ in comparison to the energies and the momentum vectors, the relation becomes

$$Q^2 = -q^2 = 2(E_\nu E_l - \mathbf{k} \cdot \mathbf{k'}).$$

- The Bjorken variable $x$ is the fraction of the target nucleon’s momentum carried by the struck quark:

$$x = \frac{Q^2}{2m_N \nu}.$$

- The fraction of the neutrino’s energy loss in the lab frame is

$$y = \frac{q \cdot p}{k \cdot p} = \frac{\nu}{E_\nu}.$$

- The Lorentz invariant Mandelstam variables are

$$s = (k + p)^2 = \frac{Q^2}{xy} + m_N^2$$

$$t = (k - k')^2$$

$$u = (k' - p)^2.$$ 

5.1.1 Neutrino nucleon scattering cross sections

A neutrino $\nu_l$ interacting with a nucleon $N$ results in a lepton $l$ of the initial neutrino flavor and several hadrons (see figure 5.1). The cross section of the inclusive charged
and neutral current interactions

$$
\nu_l N \xrightarrow{W^{(\pm)}} l X \\
\nu_l N \xrightarrow{Z^0} \nu_l X
$$

can be calculated in the parton model.

### 5.1.2 The differential cross section

In this part, an overview over the used differential cross sections for charged current and neutral current interactions will be given. To derive these cross sections the parton model is used:

- **Charged current (CC) interactions**
  For charged current interactions, the differential cross section is given as

$$
\frac{d^2\sigma}{dx\,dy} \big|_{\nu,\bar{\nu}} = \sigma_0 \cdot \left( \frac{M_W^2}{Q^2 + M_W^2} \right)^2 \left[ \left( F_2^{\nu,\bar{\nu}} \pm x F_3^{\nu,\bar{\nu}} \right) + \left( F_2^{\nu,\bar{\nu}} + x F_3^{\nu,\bar{\nu}} \right) (1 - y)^2 \right]
$$

with

$$
\sigma_0 = \frac{G_F^2 m_N E_\nu}{2\pi}.
$$

With the proton’s Parton Distribution Functions (PDFs) given above, the structure functions $F_2^{\nu,\bar{\nu}}$ and $F_3^{\nu,\bar{\nu}}$ are described as [Sch97]

- **Neutrino - Nucleon:**

$$
F_2^\nu = x \left[ u + d + s + c + \bar{t} + \bar{d} + \bar{s} \right] \quad (5.2)
$$

$$
x F_3^\nu = x \left[ u + d + 2s - \bar{t} - \bar{d} - 2\bar{s} \right], \quad (5.3)
$$

- **Antineutrino - Nucleon:**

$$
F_2^{\bar{\nu}} = x \left[ u + d + s + c + \bar{t} + \bar{d} + \bar{s} \right] \quad (5.4)
$$

$$
x F_3^{\bar{\nu}} = x \left[ u + d + 2c - \bar{u} - \bar{d} - 2\bar{s} \right]. \quad (5.5)
$$

Here, the PDFs depend on the Bjorken variable $x$ and on the momentum transfer $Q^2$.

- **Neutral current (NC) interactions**
  The differential cross section for neutral current interactions is similar to the charged current cross section:

$$
\frac{d^2\sigma}{dx\,dy} \big|_{\nu,\bar{\nu}} = \sigma_0 \cdot \left( \frac{M_Z^2}{Q^2 + M_Z^2} \right)^2 \left[ \left( F_2^{\nu,\bar{\nu}} \pm x F_3^{\nu,\bar{\nu}} \right) + \left( F_2^{\nu,\bar{\nu}} \mp x F_3^{\nu,\bar{\nu}} \right) (1 - y)^2 \right] .
$$

In NC interactions, a $Z^0$ boson is exchanged, so that the $Z$ mass appears in the propagator term instead of the $W$ boson’s mass. The structure functions are given
as

\[ F^{\nu,\nu}_2 = x \left\{ (l_u^2 + r_u^2) [u + d + 2c + \bar{u} + \bar{d} + 2\bar{c}] \right. \]
\[ + \left. (l_d^2 + r_d^2) [u + d + 2s + \bar{u} + \bar{d} + 2\bar{s}] \right\} \]
\[ x F^{\nu,\nu}_3 = x \left\{ (l_u^2 - r_u^2) [u + d + 2c - \bar{u} - \bar{d} - 2\bar{c}] \right. \]
\[ + \left. (l_d^2 - r_d^2) [u + d + 2s - \bar{u} - \bar{d} - 2\bar{s}] \right\} . \]

Here, the chiral couplings are

\[ l_u = 1/2 \cdot L_u = 1/2(1 - \frac{4}{3}x_W) \quad l_d = 1/2 \cdot L_d = 1/2(-1 + \frac{4}{3}x_W) \]
\[ r_u = 1/2 \cdot R_u = -\frac{2}{3}x_W \quad r_d = \frac{1}{3}x_W , \]

in which \( x_W = \sin^2 \theta_W \) is the weak mixing parameter which depends on the Weinberg angle \( \theta_W \).

### 5.1.3 Integration limits for the cross section

In this section, the integration limits of the \( x \) and \( y \) integration will be discussed. The integration of the differential cross section over the Bjorken variable and the neutrino's energy loss fraction is done numerically. The integration limits for the Bjorken variable \( x \) are given by the PDFLib [CER]. The model of Glück, Reya and Vogt [GRV92] has been used in all following calculations of the cross section. All models are defined within certain intervals of the Bjorken variable \( x \) and the momentum transfer \( Q^2 \). The lower boundary of the Bjorken variable is \( x > 0 \) which leads to uncertainties especially for high energies. The Model of Glück et al. has been chosen due to the low boundary of the minimal momentum transfer. The parameters in this model are

\[ x_{\text{low}} = 10^{-4} , \quad x_{\text{up}} = 0.99999 , \quad Q^2_{\text{min}} = 0.300(\text{GeV}/c)^2 , \quad Q^2_{\text{max}} = 10^8(\text{GeV}/c)^2 . \]

The neutrino's energy loss fraction does not range from 0 to 1, as it might intuitively be expected. The limits are determined by using Mandelstam analysis: The lower limit is determined by the mass of the produced lepton:

\[ y_{\text{min}} = \frac{m_l}{E_\nu} . \]

Otherwise, the energy fraction is too low for the production of a lepton. To determine the maximal energy fraction, the Mandelstam variable \( t \) is considered as:

\[ t = (p - p')^2 , \]

where the four momenta of the nucleon and the lepton are

\[ p = \left( \begin{array}{c} m_N \\ 0 \end{array} \right) , \quad p' = \left( \begin{array}{c} E_l \\ \vec{p'} \end{array} \right) . \]

Thus \( t \) can be determined to be

\[ t = m_N^2 - 2E_l m_N + E_l^2 - \vec{p'}^2 = m_N^2 - 2E_l m_N + m_l^2 . \]
The lepton energy is therefore given as

\[ E_l = -t + m_N^2 + m_l^2 \]

On the other hand, \( E_l \) can be expressed through the energy loss \( \nu = E_\nu - E_l \):

\[ E_l = E_\nu - \nu . \]

The maximum energy loss \( \nu_{\text{max}} \) is given for elastic scattering at \( \theta = 180^\circ \). In that case the Mandelstam variable \( t \) is smaller or equals zero:

\[ t \leq 0 . \]

Therefore the minimum lepton energy is given for \( t = 0 \),

\[ E_l^{\text{min}} = \frac{m_N^2 + m_l^2}{2m_N} \]

and the maximum fraction \( \nu_{\text{max}} \) is

\[ \nu_{\text{max}} = E_\nu - E_l^{\text{min}} = E_\nu - \frac{m_N^2 + m_l^2}{2m_N} . \]

The maximum energy loss fraction is therefore determined to be

\[ y_{\text{max}} = \frac{\nu_{\text{max}}}{E_\nu} = 1 - \frac{m_N^2 + m_l^2}{2m_N E_\nu} . \]

5.1.4 Total cross section

In this section, the total cross section for neutrino-nucleon scattering as well as Glashow resonance scattering of anti-electron neutrinos will be discussed.

At energies much higher than the lepton’s energy, \( E_\nu >> m_l \), the total cross section for neutrino-neutron scattering is equal for the three lepton flavors. The total cross section is shown in figure 5.2. The total cross section for anti-neutrino scattering is lower than for neutrino scattering, since the structure function which includes valence quarks is suppressed by the term \( (1 - y)^2 \). In case of neutrino scattering, only valence quark contributions contribute to the structure function which is not scaled down by the factor \( (1 - y)^2 \). The Glashow resonance of \( \nu_e e \rightarrow W \rightarrow X \) scattering is also shown in the figure. It will be examined in the following paragraph.

Glashow resonance

Neutrino-electron scattering processes are

\[ \nu_l + e^- \rightarrow W^{(\pm)} Z^0 \rightarrow \nu_l + e^- \]

with \( l = e, \mu \) and \( \tau \). The total cross sections for these processes are much smaller than for neutrino-nucleon scattering \((\sigma_{\text{tot}} \sim 10^{-42} E_\nu/ \text{GeV})\) and can thus be neglected.
However, resonant $\nu_e e^-$ scattering becomes important at energies around $E_{\nu_e} \approx M_W^2 / (2m_e) \approx 6.3$ PeV [Gla60]. The process

$$\nu_e e^- \rightarrow W \rightarrow X$$

dominates neutrino-nucleon scattering at an energy range of $3 < E_{\nu_e} / \text{PeV} < 10$ and is about 300 times larger than neutrino nucleon cross section at the resonant energy. The process is called *Glashow resonance* and its total cross section is given by [Gla60]

$$\sigma_{\text{Glashow}} \approx \frac{2G_F^2 m_e E_{\nu}}{(M_W^2 - 2m_e E_{\nu})^2 + \Gamma_W^2 \cdot M_W^2} \cdot \frac{\Gamma(W \rightarrow X)}{\Gamma(W \rightarrow e^- \nu_e)}.$$  \hspace{1cm} (5.8)

The width of the W-Boson is $\Gamma_W = 2.12$ GeV [Gro00] and $\Gamma(W \rightarrow f)$ is the inclusive branching ratio with a final state $f$. $\Gamma(W \rightarrow X)$ gives the inclusive decay width at any final state $X$. 

Figure 5.2: The total cross section versus neutrino energy. Neutrino-nucleon interactions resulting in a muon and hadrons are shown. The dashed line indicates the CC cross section and the dotted line represents the NC cross section. The Glashow resonance for $\nu_e e^-$ scattering at $\sim 6.3$ PeV is shown as well.
5.2 Production of neutrino induced muons

The neutrino induced muon spectrum is a convolution of the neutrino spectrum \( \Phi_\nu \) with the shadow factor of the Earth, \( P_{\text{shadow}} \), and the probability that the neutrino interacts with a nucleon and produces a muon visible in the detector, \( P_{\nu \rightarrow \mu} \):

\[
R_\mu(E_\mu^{\text{min}}, \theta) = \int_{E_\nu^{\text{min}}} P_{\nu \rightarrow \mu}(E_\nu, E_\mu^{\text{min}}) P_{\text{shadow}}(E_\nu, \theta) \, dE_\nu .
\]  \( (5.9) \)

The integrand includes the neutrino energy dependent effective area \( A_{\text{eff}}(E_\nu) \) as a further factor if the rate per year and steradian is calculated. Otherwise, the event rate is given in units of sr\(^{-1}\) yr\(^{-1}\) cm\(^{-2}\). In following muon rate calculations, the effective areas of AMANDA and IceCube will be included in the term \( P_{\nu \rightarrow \mu} \), since they are muon energy dependent.

5.2.1 Shadow factor

The probability that a neutrino is absorbed by the Earth due to neutrino-nucleon CC and NC interactions is given as [Leo94]

\[
P_{\text{shadow}}(X) = \exp(-X(\theta) / \lambda),
\]

in which \( X \) is the neutrino absorption length in units of cmwe and \( \lambda \) is the mean path that the particle can survive without interacting. \( \lambda \) can be written in terms of the total cross section as follows

\[
\lambda = \frac{1}{N_A \sigma_{\text{tot}}}.
\]

Here, \( N_A \) is the Avogadro constant. Subsequently the shadow factor can be expressed as

\[
P_{\text{shadow}}(X) = \exp(-N_A \sigma_{\text{tot}} X). \quad (5.10)
\]

The neutrino absorption length depends on the traveling distance through Earth. It is also dependent on the Earth’s density layers it has to pass. The traveling distance and the Earth density will be described in detail below.

Neutrino absorption

In order to calculate the absorption probability the neutrino’s path through the detector needs to be calculated. Assuming a neutrino is entering the detector under a zenith angle \( \theta \) (see figure 5.3). Then by applying the sine theorem to the triangle including \( \gamma := 180^\circ - \theta \), \( \beta \) and \( \alpha \), the relation

\[
\frac{\sin \gamma}{\sin \beta} = \frac{r_e}{r_e - d}
\]  \( (5.11) \)

is found. \( r_e \) is the radius of the Earth while \( d \) is the depth at which the detector is located under the Earth’s surface. The angle \( \alpha \) is determined as \( \alpha = 180^\circ - \gamma - \beta \). Applying the
sine theorem once more and using the equation (5.11) with the approximation \( d \ll r_e \), the distance \( d_\nu \) which the neutrino has to travel through Earth is given as\(^1\)

\[
d_\nu = \frac{\sin \alpha}{\sin \gamma} r_e .
\]  

(5.12)

The neutrino absorption length \( X(\theta) \) is the product of the density of the Earth, \( \rho_e \), with the traveling distance of the neutrino through Earth:

\[
X(\theta) = \rho_e(\theta) \cdot d_\nu(\theta) .
\]

(5.13)

\(^{1}\)Note that this is only the case for \( \sin \gamma \neq 0 \). For \( \sin \gamma = 0 \) and \( \theta = 0^\circ \), then the traveling distance of the neutrino through Earth is equal to the depth of the detector under the Earth’s surface \( (d) \):

\[ d_\nu(\theta = 0) = d . \]

If, on the other hand, \( \sin \gamma = 0 \) and \( \theta = 180^\circ \), the traveling distance is \( d_\nu(\theta) = 2 \cdot r_e - d \) since the neutrino arrives from the opposite direction.
Earth layers

Figure 5.4: Model of Earth layers. Shown is the density profile according to equ. (5.14). For simplicity, the four outermost layers as well as two inner layers these are shown as one layer, since their densities do not vary significantly (see also figure 5.5).

Figure 5.5: Density of the Earth depending on the radius according to [DA81].
5.2. Production of neutrino induced muons

Depending on the zenith angle, the neutrino has to travel through different Earth layers which have different densities. The Earth layers are shown in figure 5.4. A model of the density profile of the Earth is given by [DA81]

\[
\rho(r)[g/cm^3] = \begin{cases} 
13.0885 - 8.8381 \cdot x & x < 0.192 \\
12.5815 - 1.2638 \cdot x - 3.6426 \cdot x^2 - 5.5281 \cdot x^3 & 0.192 < x < 0.546 \\
7.9565 - 6.4761 \cdot x + 5.5283 \cdot x^2 - 3.0807 \cdot x^3 & 0.546 < x < 0.895 \\
5.3197 - 1.4836 \cdot x & 0.895 < x < 0.906 \\
11.2494 - 8.0298 \cdot x & 0.906 < x < 0.937 \\
7.1089 - 3.8045 \cdot x & 0.937 < x < 0.965 \\
2.691 + 0.6924 \cdot x & 0.965 < x < 0.996 \\
2.9 & 0.996 < x < 0.998 \\
2.6 & 0.998 < x < 0.999 \\
1.02 & x \leq 1 
\end{cases}
\] (5.14)

Here, \(x\) is given as \(x = r/r_e\) with \(r_e = 6378\) km as the radius of the Earth. Figure 5.5 shows the density of the Earth against the depth. The different layers are nearly independent of the depth. The saltuses of the function are barely significant around 6000 m. The biggest saltus of the density function is found at \(\sim 3500 m\).

The neutrino absorption length is shown in figure 5.6. It rises linearly up to \(\theta \sim 150^\circ\). At higher zenith angles, the neutrino has to travel through the Earth’s innermost core, which is much denser than other layers. Therefore, the absorption length increases more rapidly.

![Figure 5.6: Neutrino absorption length versus zenith angle.](image-url)
If one uses all components as they are described above and considers, the result for the shadow factor is shown in figure 5.7. At low energies ($E_\nu < 1$ TeV), the absorption can be neglected, while at higher energies it is $P_{\text{shadow}} < 1$. Neutrinos of $>1$ EeV are almost entirely absorbed for angles $> 90^\circ$. An observation of muon neutrinos at that energies is therefore exclusively possible at low angles, $\theta \sim 90^\circ$.

Figure 5.7: The probability that a muon neutrino (solid line) or antineutrino (dashed) survives without interacting on its way through Earth to the detector.
5.2.2 Probability of producing a muon

$P_{\nu \rightarrow \mu}$ is defined as the probability that a neutrino with an energy $E_{\nu}$ on a trajectory towards the detector produces a muon above the energy threshold of the detector ($E_{\mu}^{\text{min}}$). This probability is a convolution of the muon range $r_{\mu}$, the differential cross section of the inclusive charged current interaction $\nu N \rightarrow \mu X$, $d\sigma^{CC}/dE_{\mu}$, and the Avogadro constant $N_{A}$,

$$P_{\nu \rightarrow \mu}(E_{\nu}, E_{\mu}^{\text{min}}) = N_{A} \int_{E_{\mu}^{\text{min}}}^{E_{\nu}} dE_{\mu} \frac{d\sigma^{CC}}{dE_{\mu}}(E_{\mu}, E_{\nu}) r_{\mu}(E_{\mu}, E_{\mu}^{\text{min}}).$$  (5.15)

The lower integration limit is the minimum muon energy which can be detected and the upper integration limit is determined by the incident neutrino’s energy, since the muon cannot reach a higher energy than that. If the effective area is given dependent on the muon energy, the probability integrand is weighted with the effective area $A_{\text{eff}}(E_{\mu})$. In the following paragraph, the muon range will be discussed.

The muon range

The average muon energy loss rate in a medium is given as [GHS95]

$$\left\langle \frac{dE_{\mu}}{dX} \right\rangle = -\alpha(E_{\mu}) - \beta(E_{\mu}) \cdot E_{\mu}. \quad (5.16)$$

The first term occurs due to ionization losses of the muon. $\alpha$ can approximately be regarded as a constant with a value of $\alpha \approx 2 \text{ MeV}/(g \text{ cm}^2)$ in media like rock. The second term is due to bremsstrahlung, pair production and nuclear interactions. $\beta$ is determined with a value which is approximately constant. For rock like media, $\beta \approx 4 \cdot 10^{-6}/(g \text{ cm}^2)$.

The muon range is given by the integral of the inverse of muon loss rate over the muon energy:

$$r_{\mu} = \int_{E_{\nu}^{\text{min}}}^{E_{\nu}} \frac{1}{\left\langle dE/dX \right\rangle} dE = -\int_{E_{\mu}^{\text{min}}}^{E_{\mu}} \frac{1}{a + b \cdot E} \quad (5.17)$$

$$= \frac{1}{b} \log \frac{a/b + E_{\mu}}{a/b + E_{\mu}^{\text{min}}}. \quad (5.18)$$

When the results from above are used, the muon production probability $P_{\nu \rightarrow \mu}$ is shown for a lower muon energy $E_{\mu}^{\text{min}} = 1 \text{ GeV}$ in figure 5.8. At energies up to $\sim 1 \text{ TeV}$, the probability rises as $\sim E^{2.2}$. For higher energies, it flattens and can be approximated by a power law $\sim E^{0.8}$. 
Figure 5.8: \( P_{\nu \rightarrow \mu} \) for muon neutrino (solid lines) and antineutrino (dashed lines) interactions with matter.

### 5.2.3 Effective Areas

For the event rate calculation, it is important to consider the effective area of the detection array. The effective area is defined as the geometrical area weighted with the ratio of the detected events to the generated events. It can be determined in terms of the muon or the neutrino energy. The effective areas for the AMANDA, ANTARES and IceCube experiments are shown in figures 5.9, 5.10 and 5.11. While AMANDA’s and IceCube’s areas are dependent on the muon energy [BA03, Ice04], ANTARES effective area depends the neutrino energy [Mon03]. In each case, the effective area has been continued as a constant at energies outside of the given range.
5.2. Production of neutrino induced muons

Figure 5.9: AMANDA effective area versus muon energy according to [BA03].

Figure 5.10: ANTARES effective area [Mon03] versus neutrino energy, depending on the muon neutrino energy. The solid line is a calculation requiring an angle between the reconstructed muon and the neutrino direction of less than 1°. The dashed line is the effective area after strict reconstruction quality cuts and it will be applied in following calculations. The effective area is given in an energy range of $2 < \log(E_{\mu}/GeV) < 7$ and it has been assumed to be constant outside of this range.
5.3 Cascades

An electron neutrino which interacts with a nucleon initiates an electromagnetic or hadronic cascade. These events can be detected by large volume neutrino telescopes if the interaction takes place sufficiently close to the detection array. The event rate is the convolution of the initial neutrino spectrum with the shadow factor and the probability that a cascade was induced in the direct vicinity of the detector:

$$R_e(E_{\text{min}}, \theta) = \int_{E_{\text{min}}} P_{\nu_e \to X}(E_{\nu}, E_{\text{min}}) P^\nu_{\text{shadow}}(E_{\nu}, \theta) \, dE_{\nu}. \quad (5.19)$$

There are two main differences in comparison to the muon rate calculation. Cascades have no long track length and are usually of the order of 10 meters long. Thus an interaction has to take place within the detector and the muon range is substituted by a constant factor determined by the size of the detector (e.g. the factor is $\sim 1000$ m for IceCube). In addition, Glashow resonance of the process $\bar{\nu}_e e^- \to W^+ X$ has to be considered in cross section calculations.

The shadow factor is given as in section 5.2.1,

$$P_{\text{shadow}}(X) = \exp(-X/\lambda),$$

The behavior of the shadow factor is the same as for muon neutrinos, except for anti-electron neutrinos interacting via the Glashow resonance, the mean free path is $\lambda = 1/(10/18 \cdot N_A \cdot \sigma_{\text{tot}})$. Here, the factor $10/18 \cdot N_A$ is the number of electrons per mole water. Therefore, the shadow factor for electron neutrinos interacting with electrons of
the medium does not decrease as rapidly with the angle and the energy as it does for muon neutrinos.

5.3.1 Probability of detecting a cascade

The probability of detecting a neutrino induced cascade is the product of the neutral and charged current cross sections with the detector range,

\[ P_{\nu \rightarrow X} = (\sigma^{CC}(E_\nu) + \sigma^{NC}(E_\nu)) \cdot r_{\text{eff}}. \]

Here, \( r_{\text{eff}} \) is considered to be a constant with \( r_{\text{eff}}^{\nu_e} \approx 1000 \) m for IceCube and SALSA and \( r_{\text{eff}}^{\bar{\nu}_e} \approx 200 \) m for the AMANDA detector. It is determined by the detector size, since the interaction has to take place inside of the detector. Figure 5.12 shows \( P_{\nu \rightarrow X} \) depending on the neutrino energy. The probability increases with neutrino energy. At \( E_\nu \sim 6.3 \) PeV, a sharp peak can be observed for anti-neutrinos. This is due to Glashow resonant scattering.

![Graph showing the probability of detecting a neutrino induced cascade versus neutrino energy.](image)

Figure 5.12: Probability of detecting a neutrino induced cascade versus neutrino energy. Electron neutrino (solid lines) and antineutrino (dashed lines) interactions with matter. The size of the detector is taken to be \( \sim 1000 \) m.
5.4 Results from the event rate calculation

Muon and cascade rates will be discussed in this section. The atmospheric neutrino flux will be discussed as well as three different AGN neutrino flux models. For the detection of neutrino induced muons, three detection arrays, i.e. AMANDA, ANTARES and IceCube, will be considered. The effective areas of these experiments will be used to determine the muon rate per steradian and year. Neutrino induced cascades are calculated for the upper and the lower hemisphere.

5.4.1 Detection of muon neutrinos

In this section, the calculation of the muon event rate is shown for the conventional and the charm atmospheric neutrino flux models as well as the AGN model from Mannheim [Man95]. Furthermore, the AGN flux model from chapter 2 is discussed in two variations: The particle index is applied as it has been discussed ($p = 2.6$ for steep spectrum sources and $p = 1.5$ for flat spectrum sources) and in addition, an index of $p = 2$ is used as it is common in literature, e.g. [Man95].

The differential muon event rate is the rate without integrating over the neutrino energy. In figure 5.13 the differential rate at $90^\circ$ for a threshold energy of $E_{\mu}^{\text{min}} = 50$ GeV is shown in dependence of the neutrino energy. The flux is given in units of (GeV sr s cm$^2$)$^{-1}$. The spectrum of the atmospheric conventional neutrinos is much steeper than the one for the AGN models. The charm atmospheric flux is flatter than the conventional spectrum at lower energies ($E < 10^6$ GeV), but steepens at higher energies.

![Figure 5.13: Differential muon rate at a zenith angle of 90° versus neutrino energy at a threshold energy of 50 GeV. The solid line shows the prediction of atmospheric neutrinos, while the dotted line gives the differential rate in the AGN model of Mannheim (M $pp + p\gamma$). The dashed and dash-dotted lines are calculations in the AGN model which is discussed in chapter 2, the dashed line represents a particle index of $p = 2$, the dash-dotted line corresponds to the indices calculated from the spectral radio index of the sources.](image-url)
Figure 5.14: Event rate depending on the threshold energy of the detector. The angle is fixed at $\theta = 90^\circ$ and the chosen depth is zero.

The integral $\mu$on rate against the threshold energy is presented in figure 5.14. Again, a zenith angle of $\theta = 90^\circ$ is used. The threshold energies for various large volume neutrino telescopes (i.e. ANTARES, Baikal, AMANDA and IceCube) are indicated. By increasing the threshold energy, the background of atmospheric neutrinos can be surpassed in comparison to extragalactic signals by almost an order of magnitude. IceCube has the highest energy threshold with $E_{\mu}^{\text{min}} = 100$ GeV. The atmospheric neutrino signal is about a factor of $\sim 3$ smaller than for the ANTARES detector which has a threshold energy of $E_{\mu}^{\text{min}} = 10$ GeV.

The comparison of the rate of atmospheric neutrinos at the different threshold energies for the various detector arrays is shown against the zenith angle in figure 5.15. The detector properties are used as described in chapter 4. The atmospheric rate decreases with the zenith angle. The normalization of the rate depends on the threshold energy.

The event rate per km$^2$·yr and steradian for the AMANDA detector is shown in figure 5.16 as a function of the zenith angle. The same calculation for ANTARES, Baikal and IceCube properties is presented in appendix B.2.3. The angle dependence is similar for the different detector arrays: The AGN model predictions and the charm flux calculation vary little with the zenith angle. The dominant contribution to the rate is related to the lower integration limit where the shadow factor is $P_{\text{shadow}} \sim 1$. The atmospheric rate decreases slightly with the zenith angle as the input spectrum decreases with the angle.
Figure 5.15: Atmospheric rate of neutrino induced muons versus zenith angle for various neutrino telescopes: AMANDA (solid), ANTARES (dashed), Baikal (dotted) and IceCube (dash-dotted).

Figure 5.16: Event rate versus zenith angle for AMANDA detector properties. Units are (s sr cm²)⁻¹. The effective area of the detection array has not been taken into account yet. The flux models are as in figure 5.13.
Figures 5.17, 5.18 and 5.19 show event rates per year and steradian for AMANDA, ANTARES and IceCube as a function of $\theta$. The effective areas are used as described in chapter 4. For ANTARES and AMANDA, the atmospheric spectrum results in approximately $10^3$ events per year and steradian. Two AGN models ($M(p_\nu + p_\gamma)$ and Becker ($p = 2$)) produce a rate of $\sim 10$ events. The charm model prediction lies an order of magnitude below. ($\sim 1$ event). The model from chapter 2 is contributing with a flux of $\sim 0.1$ events, since the normalization of the flux is very low. If IceCube’s detector properties are used, where the effective area is at least $0.6 \text{ km}^2$, the rates are each an order of magnitude higher than for AMANDA and ANTARES. Two of the AGN models are predicted to contribute with more than 100 events per year and steradian. That means that the integral rate is such that an extraterrestrial contribution to the total spectrum should be detectable with IceCube. Even with the AMANDA and ANTARES detector, an observation of the extragalactic component seems to be possible with a data taking phase of some years.

![Graph](image-url)

Figure 5.17: AMANDA detector rate per year and steradian versus zenith angle. AMANDA’s effective area (see figure 5.9) and muon threshold energy ($E^\text{min}_\mu = 50 \text{ GeV}$) have been used.
Figure 5.18: ANTARES detector rate per year and steradian versus zenith angle. ANTARES’ effective area (see figure 5.9) and muon threshold energy ($E_{\mu}^{\text{min}} = 10$ GeV) have been used.

Figure 5.19: IceCube detector rate per year and steradian versus zenith angle. IceCube’s effective area (see figure 5.11) and muon threshold energy ($E_{\mu}^{\text{min}} = 100$ GeV) have been used.
5.4.2 Detection of electron neutrinos

The event rate for electron neutrino induced cascades will be discussed in this section, taking into account the conventional atmospheric electron neutrino spectrum and the three AGN models which have already been discussed in section 5.4.1. The differential rate, depending on the neutrino energy is shown in figure 5.20. The atmospheric spectrum is again steeper than all three AGN predictions. A peak at $\sim 6.3$ PeV is due to Glashow resonance scattering. Cascades can also be observed from above, since there is no dominating background as it is the case of muon observations.

![Figure 5.20](image.png)

Figure 5.20: Differential cascade rate per energy, steradian, year and square kilometer dependent on the neutrino energy. The atmospheric neutrino flux is calculated (solid line) as well as three AGN neutrino flux models: The dotted line is based on the blazar model from Mannheim [Man95]. The dashed line is calculated by using the result for flat ($p = 1.5$) and steep ($p = 2.6$) spectrum sources from chapter 2 (see figure 2.22). The dash-dotted line represents a calculation for the model from chapter 2 (see figure 2.23) with the particle index $p = 2$ for flat and steep spectrum sources.
Figure 5.21 shows the atmospheric rate integrated over 4π and the calculation for the AGN model from chapter 2 with calculated particle indices, integrated over the lower hemisphere (dotted), over the upper hemisphere (dashed) and the sum of both. The same spectra are shown for the two other AGN models (Becker (p = 2) and M(pp+pγ)) in figures 5.22 and 5.23. The advantage of looking upwards can clearly be seen at higher energies. The rate for the upper hemisphere shows an access of events around \( \sim 6.3 \) PeV due to Glashow resonance scattering, while neutrinos are absorbed in the Earth by the same effect when looking downwards.

Figure 5.21: Differential cascade rate per year, square kilometer and energy. The atmospheric rate, shown as the solid line, is compared to the neutrino flux model from chapter 2. The dotted line represents the rate from the lower hemisphere and the dashed line shows the calculation from the upper hemisphere.
Figure 5.22: Differential event rate for atmospheric neutrinos and neutrinos from AGN as calculated in chapter 2. A particle index of $p = 2$ is assumed. The dotted line represents the rate from the lower hemisphere and the dashed line shows the calculation from the upper hemisphere.

Figure 5.23: Atmospheric differential event rate and rate according to the AGN flux model from Mannheim [Man95]. The dotted line represents the rate from the lower hemisphere and the dashed line shows the calculation from the upper hemisphere.
The integral event rate, depending on the threshold energy is shown in figure 5.24. An effective range of 1000 m has been assumed, representing the size of the IceCube detector. The event rate is directly proportional to the effective range. In case of AMANDA, the rate would be a factor of five smaller when an effective range of 200 m is assumed. With a threshold energy of $E_{\nu}^{\text{min}} \sim 10^{4.7}$ [Kow04], two of the AGN models are about two orders of magnitude higher than the atmospheric spectrum. The third AGN model is of the same order as the atmospheric spectrum. The number of AGN neutrinos for the two AGN models is more than 200 events per year and km$^2$. The atmospheric flux and the third AGN model would only result in one event per year and km$^2$.

![Diagram](image)

Figure 5.24: Event rate for electron neutrinos, varying with the threshold energy of the detector. Models as in figure 5.20. The threshold energy for AMANDA, $\log(E_{\nu}^{\text{min}}) = 4.7$ is indicated [Kow04].

Cascade-like events are also observable with a detector in a large salt mine as the potential future experiment SALSA is planned to be built. If one assumes a 1 km$^2$ detector, the event rate can be increased about a factor of 2.2 compared to IceCube, since the rate is proportional to the total neutrino-nucleon cross section and salt is about a factor of 2.2 denser than water.

On the whole, the observation of cascade like events seems to be a very promising field for the detection of neutrinos from Active Galactic Nuclei. Even if the statistics with present day neutrino telescopes are still low, neutrino detection arrays like IceCube will presumably be able to observe an extragalactic component of the electron neutrino flux.
5.4. Results from the event rate calculation
Chapter 6

Conclusions

In this thesis two main things have been examined. The first question to be answered
was how many neutrinos are there actually arriving at Earth from Active Galactic
Nuclei. The second leading question is, how many of these events can be observed
in a large volume neutrino detector and is the signal contingently suppressed by the
atmospheric neutrino flux.

In the first part of this thesis, a new model of the neutrino flux from Active Galactic
Nuclei has been developed. The generic AGN spectrum has been developed using the jet-
disk symbiosis model [FMB95]. Additionally, the maximum proton energy was assumed
to be luminosity dependent. The particle index of the power law energy spectrum has
been determined from the spectral indices of the used sources at 5 GHz. Both steep and
flat spectrum sources have been taken into account [W+00, Pea85]. The cosmological
parameters have consistently been used as (\(\Omega_m = 1\), \(\Omega_\Lambda = 0\), \(h = 0.5\)). The result of the
calculation seems to be independent of the choice of cosmological parameters within the
errors of the used functions as it has been examined in chapter 3. The resulting spectrum
is dominated by steep spectrum sources up to energies of \(10^6\) GeV. At higher energies,
the flat spectrum sources contribute more than the steep spectrum sources. If an index
of \(p = 2\) is assumed for the whole spectrum, the flux is about \(\Phi \cdot E^2 \approx 10^{-7}\) GeV/(s sr
\(\text{cm}^{-2}\)) for one neutrino flavor. This is about an order of magnitude below the limits set
by the AMANDA experiment, which are determined using an input spectrum of \(E^{-2}\).

In the second part of this thesis, the neutrino induced event rate has been determined
for several large volume neutrino experiments. The charged and neutral current cross
sections of neutrinos with nucleons have been determined using the PDFLib in the model
of Glück et al. [GRV92]. For electron neutrino interactions, the Glashow resonance at
\(\sim 6.3\) PeV has been taken into account. The neutrino induced muon event rate has been
evaluated for the lower hemisphere only, since the signal is suppressed by atmospheric
muons when looking upwards. The effective areas of three experiments, i.e. AMANDA,
ANTARES and IceCube, have been used to determine the neutrino induced muon rate
per year and steradian. AGN neutrino rates are about two orders of magnitude lower
than the atmospheric rate. For detector arrays like AMANDA and ANTARES, the
number of muons per year and steradian from the atmosphere is about thousand, while
it is approximately 10 for AGN models. The atmospheric neutrino flux from baryons
with charm contribution would only be a negligible background to the whole spectrum.
(\sim 1 \text{ event per year and steradian}). In a detection array like IceCube, the rate can be increased by about a factor of ten. That implies that the AGN neutrino rate is of the order of \(10^{0.5}\) events. Much higher energy thresholds apply for the detection of cascades. For instance the threshold energy for the detection of neutrino induced muons in the AMANDA detector is \(\sim 50 \text{ GeV}\) while the threshold for cascades is \(\sim 10^{4.7} \text{ GeV}\) [Kow04]. Cascades can be observed from the upper hemisphere. At energies \(> 10^{5} \text{ GeV}\) the AGN flux is much higher from above than from below. The atmospheric electron neutrino flux is lower than the AGN neutrino flux at that energies if a particle index of \(p = 2\) is used. However, it is of the same order of magnitude if the index is calculated from the sources spectral index. It can be concluded that the integral AGN neutrino contribution to the whole spectrum is some orders of magnitude higher than the spectrum of atmospheric charm baryons if models with \(p = 2\) apply. Detection arrays of the size of IceCube should be able to detect an extragalactic signal.
Schlußfolgerungen


Im zweiten Teil dieser Arbeit wurde die neutrinoinduzierte Ereignisrate für verschiedene, großvolumige Neutrinoexperimente berechnet. Die CC- und NC-Wirkungsquerschnitte von Neutrinos mit Nukleonen wurden mit Hilfe der PDFLib unter Verwendung des Modells von Glück et al. bestimmt [GRV92]. Für die Elektronneutrino-Berechnungen musste zusätzlich die Glashow Resonant bei $\sim 6.3$ PeV beachtet werden. Die Ereignisrate für neutrinoinduzierte Myonen wird ausschließlich für die untere Hemisphäre berechnet, da der Fluß aus der oberen Hemisphäre von atmosphärischen Myonen überdeckt wird. Die effektiven Flächen von verschiedenen Experimenten, insbesondere von AMANDA, ANTARES und IceCube, sind verwendet worden, um die neutrinoinduzierten Myonrate pro Jahr und Steradian zu bestimmen. Für Detektoren wie AMANDA und ANTARES erhält man ca. 1000 Ereignisse pro Jahr und Steradian, während man aus den AGN-Modellen jeweils ca. 10 Ereignisse erhält. Der aus Charginquark beinhaltenen Hadronen resultierende atmosphärische Neutrino-Fluss trägt nur etwa ein Ereignis bei. Die Rate kann durch eine Vergrößerung des Detektorvolumens erhöht werden, was bei IceCube der Fall ist. Im Endausbau umfaßt IceCube ein Volumen von 1 km³ und die Rate kann so etwa um einen Faktor 10 erhöht werden. Das bedeutet, daß die AGN-Neutrino-Fluss von der Größenordnung $10^2$ ist. Für die Detektion von neutrinoinduzierten Kaskaden ist die Energieschwelle höher als bei der Beobachtung von neutrinoinduzierten Myonen. Beispielsweise liegt die Schwellenergie im AMANDA-Detektor für Myonen bei 50 GeV, während sie sich für Kaskaden bei einem Wert von $\sim 10^{-3}$ GeV befindet [Kow04]. Kaskaden können besonders gut beobachtet werden, wenn sie von der oberen Hemis-
Slutsatsen

Två huvudaspekter undersöcktes i denna uppsats. Första frågan var hur många neutriner som effektivt når jorden från Aktiva Galaktiska Kärnor. Den andra ledande frågan var hur många händelser som egentligen detekteras av neutrino-teleskop och om signalen är signifikant jämfört med händelsetalet från det atmosfäriska neutrinospektret.

I uppsatsens första del utvecklades en ny modell för neutrinoflödet från Aktiva Galaktiska Kärnor. Det generiska AGN spektrum bestämdes med hjälp av ”jet-disk symbiosis” modellen [FMB95]. Dessutom antogs att den maximala partikel energin beror på AGN skivans luminositet [Lov76]. Energispektrets index beräknades genom att använda relationen till de spekrala indexen från källans synkrotronstråling. Hänvis togs till både brant- och slätspetrumfläckar. Kosmologiska parametrar användes inom Einstein-de Sitter kosmologin ($\Omega_m = 1$, $\Omega_{\Lambda} = 0$) med $\Omega = 0.5$. Resultatet verkar vara oberoende av valet av de kosmologiska parametrarna som visades i kapitel 3. Up till $E \sim 10^6$ GeV domineras det resulterande spektret av brantspetrumfläckor. Vid högre energi är det huvudsakligen slätspetrumfläckor som producerar spektrum. När indexet $p = 2$ används är det resulterande flödet $\Phi \cdot E^2 \approx 10^{-7}$ GeV/(s sr cm$^2$) för en neutrinosort. Detta värde ligger ungefär en storleksordning under de experimentella gränserna som ges från AMANDA experimentet. Dessa gäns beräknades genom att anta ett $E^{-2}$ spektrum.

Appendix A

Tables

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Table A.2: 171 flat spectrum sources and their spectral index at 5 GHz.
Appendix B

Overview of the calculation results

B.1 Neutrino flux from Quasars

A summary of all the functions used for the calculation of the isotropic AGN neutrino flux is given in this section.

B.1.1 Steep spectrum sources

- The generic energy spectrum

\[
\frac{dN}{dE_\nu} = \phi_0 \cdot E_\nu^{-p} \exp \left[ - \frac{E_\nu}{E_{\text{max}}} \right]
\]

with a normalization constant

\[
\phi_0 = \frac{x \cdot L_{\text{disk}}}{p - 2 \left[ \frac{E_{\text{min}}}{E_\nu} \right]^{2-p} + f(p) \cdot \left[ \frac{E_{\text{max}}}{E_\nu} \right]^{2-p}}
\]

\( p \) is the neutrino spectral index which is determined to be 2.6 in section 2.1 and

\[
f(p) = \frac{1}{2 - p} - \frac{1}{3 - p} + \frac{1}{2} \cdot \frac{1}{4 - p} - \frac{1}{6} \cdot \frac{1}{5 - p} + \frac{1}{24} \cdot \frac{1}{6 - p}.
\]

\( L_{\text{disk}} \) is given by the jet-disk symbiosis:

\[
L_{\text{disk}} = 4.29 \cdot 10^{45} \cdot (1 + z)^{-1/2} \left( L_{42}^{\text{ext}} \right)^{2/3}.
\]

(B.1)

The maximum energy depends on the luminosity as

\[
E_{\text{max}} = 10^{10} \cdot (1 + z)^{-1/4} \cdot L_{42}^{1/3}
\]

- The radio luminosity function by Willott et al. is a product of a luminosity depending part and a redshift dependent evolution function [W+00]:

\[
\frac{dn}{dL}(L, z) = \frac{dn}{dL}(L) \cdot f(z).
\]
The pure luminosity function consists of a low luminosity function (sources with weak emission lines) and a high luminosity function (strong emission line FR-II galaxies). The low luminosity function is given as

\[
\frac{dn}{dL_{\text{low}}} (L) = \frac{\rho_0}{L \cdot \ln(10)} \left( \frac{L}{L_0} \right)^{-\alpha_l} \cdot \exp \left[ -\frac{L}{L_0} \right].
\]

while the high luminosity part can be described as

\[
\frac{dn}{dL} (L) \bigg|_{\text{high}} = \frac{\rho_0}{L \cdot \ln(10)} \left( \frac{L}{L_h^*} \right)^{-\alpha_h} \cdot \exp \left[ -\frac{L}{L_h^*} \right].
\]

The parameters used are given in tables 2.1 in section 2.2.2.

### B.1.2 Flat spectrum sources

- The generic spectrum is, as for steep spectrum sources, given as

\[
\frac{dN}{dE_\nu} = \phi_0 \cdot E_\nu^{-p} \exp \left[ -\frac{E_\nu}{E_{\text{max}}} \right]
\]

with a normalization constant of

\[
\phi_0 = \frac{x \cdot L_{\text{disk}}}{f(p) \cdot \left[ \frac{E_{\text{max}}}{\text{GeV}} \right]^{2-p}}.
\]

\( f(p) \) is given as for the steep spectrum sources while the jet-disk symbiosis for compact cores is

\[
L_{\text{disk}} = 2.12 \cdot 10^{45} \left( L_{42}^{\text{comp}} \right)^{1.27} \left[ \frac{\text{erg}}{s} \right].
\]

The maximum energy is given as

\[
E_{\text{max}} = 7.5 \cdot 10^9 \cdot L_{42}^{0.3935}
\]

- Dunlop et al. have a pure luminosity evolution ansatz for the RLF of flat spectrum AGN which is

\[
\frac{dn}{dL}(L,z) = \frac{\rho_0}{\ln(10) \cdot L} \cdot \left\{ \left( \frac{L}{L_c(z)} \right)^{\alpha} + \left( \frac{L}{L_c(z)} \right)^{\beta} \right\}^{-1},
\]

with

\[
L_c(z) = 10^{a_0 + a_1 \cdot z + a_2 \cdot z^2}.
\]

The free parameters are given in table 2.2 in section 2.2.3.
Appendix B. Overview of the calculation results

B.1.3 Cosmology

- The comoving volume divided by $4\pi d_L^2$, where $d_L$ is the luminosity distance, is given in units of Gpc$^{-3}$·cm$^{-2}$:

$$\frac{dV_c/dz}{4\pi d_L^2} = \frac{3.15 \cdot 10^{-15}}{(1+z)^2} \cdot \left[ (1+z)^2 \cdot (1 + \Omega_m \cdot z) - \Omega\Lambda \cdot z \cdot (2 + z) \right]^{-1/2} \left[ \frac{\text{Gpc}^3}{\text{cm}^2 \cdot \text{s} \cdot \text{yr}} \right].$$

- Due to adiabatic energy losses, the neutrino energy at the detector, $E_\nu^0$, is smaller than the particle’s energy at the AGN, $E_\nu$:

$$E_\nu = (1 + z) \cdot E_\nu^0.$$
B.2 Detector Rate

B.2.1 Cross sections

The amplitude $\mathcal{M}$ of the processes

$$
\nu_l N \xrightarrow{W^{(\pm)}} l X
$$

$$
\nu_l N \xrightarrow{Z^0} \nu_l X
$$

is given by

$$
\mathcal{M} = \sqrt{2} G_F \, \overline{\psi}(k') \gamma^\mu (1 - \gamma_5) u(k) \left( 1 + Q^2 / M_W^2 \right)^{-1} \left\langle X | J^\text{weak}_\mu (0) | p, \sigma \right\rangle .
$$

Here,

$$
\overline{\psi}(k') \gamma^\mu (1 - \gamma_5) u(k)
$$

is the leptonic contribution with $\overline{\psi}(k')$ as the outgoing fermion spinor with a momentum $k'$ and $u(k)$ as the ingoing fermion spinor of momentum $k$. $\gamma^\mu$ are the Dirac matrices and $1/2(1 - \gamma_5)$ is the projection operator onto left-handed spinors [PS95].

$$
(1 + Q^2 / M_W^2)^{-1}
$$

is the electroweak propagator of the $W$ (charged current) or $Z$ (neutral current) boson with $M_W = 80.419$ GeV and $M_Z = 91.1882$ GeV as the bosons’ masses [Gro00]. The term

$$
\left\langle X | J^\text{weak}_\mu (0) | p, \sigma \right\rangle
$$

is the hadronic contribution. Here, $\langle X \rangle$ is the final hadron state and $J^\text{weak}_\mu$ is the electroweak current. $|p, \sigma\rangle$ is the state of the initial nucleon with a momentum $p$ and a spin configuration $\sigma$. $G_F = 1.166 \cdot 10^{-5}$ GeV$^{-2}$ is Fermi’s constant.

For charged current interactions, the electroweak current is a pure V-A vector with

$$
J^{CC}_\mu = \overline{\psi}(x) \gamma^\mu (1 - \gamma_5) \left[ d(x) \cdot \cos(\theta_c) + s(x) \sin(\theta_c) \right] + c(x) \gamma^\mu (1 - \gamma_5) \left[ s(x) \cdot \cos(\theta_c) - d(x) \right],
$$

where $\theta_c = 13^\circ$ is the Cabibbo angle. $u, d, s, c$ and $\overline{\psi}$ are parton distribution functions (PDFs) for the protons.

Parton distribution functions

The parton distribution functions (PDFs) describe the structure of the proton. Figure B.1 shows a set of parton distribution functions for the proton. The proton consists of $uud$ as constituents, quark-antiquark pairs (sea quarks) and gluons. The contribution of the gluons and the sea quarks increases as the Bjorken variable decreases.

If the distribution functions of the proton are known, the corresponding functions for the neutron can be concluded: A neutron consists of $udd$ constituents. If $q = u, d, s, c, b, t$
are the proton’s distribution functions (this convention will be used throughout this thesis), the neutron’s distribution functions $q^n$ obey

\[ u^n(x) = d(x), \quad d^n(x) = u(x), \quad \bar{u}^n(x) = \bar{d}(x) \quad \text{etc.} \quad (B.2) \]

The PDFs of the proton are measured in several experiments and are listed in [CER]. The PDFs $u$ and $d$ consist of a valence and a sea quark contribution, $u = u_v + u_s$ and $d = d_v + d_s$, while $s, c, b$ and $t$ as well as $\bar{u}, \bar{d}, \bar{s}, \bar{c}, \bar{b}$ and $\bar{t}$ are pure sea quark contributions. The neutral current is given in the Weinberg Salam Glashow theory as [Rob90]

\[ J^{NC}_\mu = \sum_{q = u, d, s, \ldots} \bar{q}(x) \gamma_\mu [ (L_q + R_q) - (L_q - R_q) \cdot \gamma_5 ] q(x). \]

$L_q$ and $R_q$ are the chiral couplings with $q = u, d, s, c, b, t$ given in terms of the Weinberg angle $\theta_W$. Introducing the weak mixing parameter $x_W := \sin^2(\theta_W) = 0.226$, the chiral couplings can be expressed as

\[ L_u = 1 - \frac{1}{3} x_W \quad L_d = -1 + \frac{2}{3} x_W \]
\[ R_u = -\frac{1}{3} x_W \quad R_d = \frac{2}{3} x_W. \]

The differential cross section is given by

\[ d\sigma = \frac{1}{\sqrt{\nu^2 + Q^2}} \frac{d^3k'}{2k'_0} \frac{1}{4} \sum_{\sigma} |M|^2. \]

It can be written as the product of the leptonic and the hadronic tensor, $l_{\mu\nu}$ and $W_{\mu\nu}$:

\[ k'_0 \frac{d\sigma}{d^2k'} = \frac{M}{s - M^2} \frac{C_F^2}{(4\pi)^2(1 + Q^2/M_W^2)^2} l_{\mu\nu} W_{\mu\nu} \quad (B.3) \]

The lepton and the hadronic tensor depend on the current vector.
B.2.2 Calculation

- **Muon neutrinos**

\[ R_\mu(E_\mu^{\text{min}}, \theta) = \int_{E_\mu^{\text{min}}}^{E_\mu} P_{\nu \rightarrow \mu}(E_\nu, E_\mu^{\text{min}}) P_{\text{shadow}} \Phi_\nu(E_\nu, \theta) \, dE_\nu. \]

with

\[ P_{\text{shadow}}(X) = \exp(-N_A \sigma_{\text{tot}} X). \]

as the Earth shadow factor (\(N_A\): Avogadro constant, \(\sigma_{\text{tot}}\): total cross section and \(X\): Neutrino absorption length).

\[ P_{\nu \rightarrow \mu}(E_\nu, E_\mu^{\text{min}}) = N_A \int_{E_\mu^{\text{min}}}^{E_\nu} \frac{d\sigma^{CC}}{dE_\mu}(E_\mu, E_\nu) R_{\text{eff}}(E_\mu, E_\mu^{\text{min}}) \]

is the probability of producing a muon. Here, \(E_\mu^{\text{min}}\) is the threshold energy of the muon. \(R_{\text{eff}}\) is the effective range of the muon. \(\Phi_\nu(E_\nu, \theta)\) is the generated neutrino flux.

- **Electron neutrinos**

\[ R_e(\theta) = \int_{E_{\mu}^{\text{min}}}^{E_e} P_{\nu \rightarrow e}(E_e) P_{\text{shadow}}(\theta) \Phi_\nu(E_e, \theta) \, dE_e. \]

\(\Phi_\nu\) and \(P_{\text{shadow}}\) are as for muon calculations, except that when concerning the Glashow resonance cross section, \(N_A\) has to be multiplied with a factor 10/18 in the shadow factor, since neutrinos interact with electrons on their way through Earth. The probability of producing a cascade within the detector is

\[ P_{\nu_e \rightarrow X} = (\sigma^{CC}(E_\nu) + \sigma^{NC}(E_\nu)) \cdot R_{\text{eff}}^{\nu_e}. \]

Cascades can be induced by charged current and neutral current interactions. The effective range is given by the size of the detector.

B.2.3 Figures of event rates for various detector arrays
Appendix B. Overview of the calculation results

![Graph showing results for different models and energies.](image)

- For $d=1750$ m, $E_{T}^{*}=50$ GeV (AMANDA):
  - Atmospheric
  - M (pp+γ)
  - Becker (p=2)
  - charm

- For $d=2225$ m, $E_{T}^{*}=10$ GeV (Antares):
  - Atmospheric
  - M (pp+γ)
  - Becker (p=2)
  - Becker
  - charm
B.2. Detector Rate

- For $d=1070$ m, $E_{\nu} = 15$ GeV (Baikal)
- $\nu (p+p)$
- Becker ($p=2$)
- Becker
- charm

- For $d=1900$ m, $E_{\nu} = 100$ GeV (IceCube)
- atmospheric
- $\nu (p+p)$
- Becker ($p=2$)
- Becker
- charm
B.3 Physical Constants

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<tr>
<td>Cabbibo angle ( \theta_c = 13^\circ )</td>
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</table>

Astronomy

| Earth radius \( R_{\oplus} = 6378 \text{ km} \) |

Cosmology

| Hubble parameter \( h = 0.71 \) |
| Matter density \( \Omega_m = 0.27 \) |
| Dark matter \( \Omega_{\Lambda} = 0.75 \) |

Anmerkung

Danksagung

Viele Leute haben einen sehr großen Beitrag dazu geleistet, daß diese Arbeit realisiert werden konnte. An dieser Stelle möchte ich allen direkt und indirekt Beteiligten für ihre Hilfe danken.

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Erklärung

Hiermit versichere ich, daß ich diese Arbeit selbständig verfaßt und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt sowie Zitate kenntlich gemacht habe.
Bibliography


[Sal] D. Saltzberg. Private communication. UCLA.


