Implications of optical properties of ocean, lake, and ice for ultrahigh-energy neutrino detection

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The collecting power and imaging ability of planned ultrahigh-energy neutrino observatories depend on wavelength-dependent absorption and scattering coefficients for the detector medium. Published data are compiled for deep ice at the South Pole, for deep fresh water at Lake Baikal, and for deep seawater. The effective scattering coefficient is smallest for the clearest deep ocean sites, whereas the absorption coefficient is an order of magnitude smaller for deep ice than for the ocean and lake sites. The effective volume per detector element as a function of energy is calculated for electromagnetic cascades produced by electron neutrinos interacting at the various sites. It is largest for deep bubble-free ice, smallest for shallow bubbly ice, and intermediate for lake and seawater. The effective volume per element is calculated for detection of positrons resulting from the capture of a few megaelectron volt supernova neutrinos by protons in the medium. This volume is proportional to the absorption length and independent of the scattering length; it is larger for ice than for seawater or lake water. © 1997 Optical Society of America

1. Introduction

There is great international interest in designing and constructing observatories optimized for detecting high-energy neutrinos from astrophysical sources such as active galactic nuclei.1 All agree that such observatories will require a huge volume of transparent, deep material such as lake or ocean water or ice, which acts as both the target and the medium for detecting the charged particles produced in interactions of such neutrinos. Estimates of the fluxes and detection rates of ultrahigh-energy astrophysical neutrinos lead to the requirement of an effective volume >1 km³ so as to see at least a few events per day. Two instruments consisting of three-dimensional arrays of phototubes have made encouraging starts. The first is an array of five strings with a total of 23 working pairs of phototubes at a depth of ~1 km in Lake Baikal.2 The second is the Antarctic muon and neutrino detecting array (AMANDA) South Pole array, consisting of four strings with 73 working phototubes frozen into ice at depths of 0.8–1 km (Ref. 3), below which are five new strings with 79 working phototubes at depths of 1.6–2 km (Ref. 4). Both of these use solid ice as a platform from which to deploy strings. (For Baikal, the platform is solid only during the Russian winter; for each AMANDA string, a hot-water drill is used to melt ice to the desired depth; the water in the hole refreezes after the string is in place.) For deep ocean sites, such as have been proposed for the deep underwater muon and neutrino detector (DUMAND)5 and NESTOR,6 the problems of deploying strings from shipboard have not been solved, but the basic concept is the same: The phototubes of an array accurately record arrival times and intensities of the photons in the moving cone of Cherenkov light produced by muons or electrons created in interactions of muon or electron neutrinos, respectively.

The optical properties of the medium become of crucial importance when one takes into account the cost of expanding from the small-scale instruments now under construction to km³ scale volumes. The region of interest for detection of Cherenkov light is 320 ≤ λ ≤ 620 nm. At wavelengths shorter than ~320 nm, light is absorbed by the glass pressure vessel that houses a phototube; at wavelengths longer than ~620 nm, the quantum efficiency of the phototube becomes too low. The quantities relevant to the media are the wavelength-dependent absorption and scattering lengths and the angular distribution of scattered photons. At the sites proposed for DUMAND (Pacific Ocean near Hawaii) and NESTOR (Mediterranean Sea near Greece), absorption was measured only at one wavelength, whereas attenua-
tion was measured at a number of wavelengths and depths. For those cases, I estimated the absorption spectra by assuming a wavelength-dependent model of scattering and subtracting scattering from attenuation as described in Subsection 3.B, then calculated effective volumes per detector module at various sites for the detection of electromagnetic cascades from the interaction of electron neutrinos as a function of energy and for the detection of neutrinos with a mean energy of 20 MeV emitted from a supernova. In this paper I do not attempt to estimate effective volumes for detection of muons from interaction of muon neutrinos. For that case, reconstruction of the muon's trajectory is of interest, and this is strongly affected by the scattering coefficient, which is either poorly known (for deep ocean waters) or varies with depth (South Pole ice).

Scaling up from an effective volume of a single module for detection of a single photon to an effective volume of an optimally spaced array of many modules is also not discussed. Roughly, \( V_{\text{array}} \approx \frac{N_{\text{PMT}}}{F} \), where PMT is the photomultiplier tube, \( N_{\text{PMT}} \) is the total number of modules, and \( F \) is a fraction that takes into account the experimental requirement that the event be recorded in many modules so as to reject background and to measure energy or direction. Typically, \( F \) might be of the order of or somewhat less than 0.1 for the detection of a \( v_e \)-induced cascade or a \( v_\mu \)-induced muon.

2. Optical Properties of Hypothetical Pure Fresh Water and Seawater

A standard reference is Smith and Baker.\(^7\) For wavelengths from 200 to 800 nm, they compiled a table giving values, for idealized pure seawater, of the diffuse attenuation coefficient \( K_d(\lambda) \), the absorption coefficient \( a_w(\lambda) \), and the molecular scattering (Einstein–Smoluchowski) coefficient \( b_w(\lambda) \). In compiling this table, they used values of \( K_d(\lambda) \) measured in the clearest ocean water (the Sargasso Sea), but only at shallow depths where sunlight penetrates. Using the expression \( a_w(\lambda) = K_d(\lambda) - 0.5b_w(\lambda) \), based on radiative transfer theory, they obtained an upper bound on \( a_w(\lambda) \). They assumed that the concentration of dissolved and suspended particulate material was too low to contribute to scattering. Their values for molecular scattering are assumed to be reliable in the limit of negligible scattering by particulates and dissolved impurities. None of the natural ocean and lake sites under consideration for high-energy neutrino astronomy is as clear as the hypothetical water of Smith and Baker, although water near the ocean floor at the DUMAND and NESTOR sites comes close.

Here we are not concerned with \( K_d(\lambda) \), which is defined for natural lighting and measures the penetration of radiant energy in ocean water, corrected for the Sun directly overhead. The quantities of interest for both natural water and natural ice are \( a_w(\lambda) \) and \( b_w(\lambda) \). Measurements in the literature are most often based on loss of intensity of light in a collimated beam that is due to both absorption and scattering out of the beam, and values are given for the attenuation coefficient \( c_w = a_w + b_w \). To obtain absorption, one often uses an empirical model to subtract the contribution of scattering (either for fresh water or seawater, as appropriate) from the attenuation. Figure 1 compares data for attenuation coefficients as a function of wavelength for water from various sources, including specially purified fresh water. In Section 3 the method for correcting for scattering is discussed.

The open circles in Fig. 1 show the data of Quickenden and Irvin,\(^9\) who hold the record for the purest and clearest fresh water. They found that, in the wavelength interval from 196 to 320 nm, repeated steps of purification led to a monotonic decrease in the attenuation, eventually reaching asymptotic values. When corrected for molecular scattering (calculated) so as to give absorption \( (a_w = c_w - b_w) \), their absorption coefficients for purest water are an order of magnitude lower than those compiled by Smith and Baker throughout the region \( \lambda = 220–320 \) nm. Nearly as good as the Quickenden and Irvin water was the purified fresh water studied by Boivin et al.,\(^9\) intended for use in the Sudbury Neutrino Observatory. They emphasized that impurities increase both absorption and scattering. Their data are shown as solid circles with error bars in Fig. 1. From the observed proportionality of their attenuation to \( \lambda^{-4} \) at short wavelengths, they concluded that their higher values for attenuation coefficient than those of Quickenden and Irvin at short wavelengths were probably due to Rayleigh scattering from a colloidal suspension of subwavelength size. Thus the absorption data of Boivin et
must be regarded as upper limits for pure fresh water.

3. Optical Properties of Lake Baikal and Ocean Water at DUMAND and NESTOR Sites

The great majority of published research on the spectral distribution of attenuation and absorption coefficients for natural water applies to optical properties in the uppermost 100 m of seas or lakes where sunlight penetrates and where life is concentrated. At depths of thousands of meters in the ocean that are of interest to high-energy neutrino astrophysics, few data on attenuation, absorption, and scattering as a function of wavelength have been reported in the refereed literature. A high-energy neutrino observatory in the ocean would probably extend upward from a depth of \( \sim 10^2 \) m above the ocean floor to a maximum height of \( \sim 1 \) km above the ocean floor.

A. Lake Baikal Site

Belolaptikov et al.\(^{10,11}\) used two methods to study optical properties of Lake Baikal at a depth of \( \sim 1000 \) m where their array is deployed. In Ref. 10 they reported measurements of the absorption coefficient at wavelengths from 375 to 550 nm for June and November 1993. The seasonal change in absorption was noticeable but small. They found large variations of the scattering length that were due to changing currents of biomass and particulates transported from the upper layers of the lake downward. This sedimentation caused a decrease of PMT sensitivity of approximately 50% within approximately 140 days. In Ref. 11 they used a pulsed laser technique to measure optical properties at one wavelength, 475 nm. They found an absorption length of 20 \( \pm 2 \) m and a value of scattering length of roughly 10 m (with a large error).

Recently, Pandel\(^{12}\) analyzed all the Baikal data taken at 475 nm, including laboratory measurements of the angular distribution of scattered light in samples collected in bottles. For the absorption length, defined as \( \lambda_a = 1/a \), he obtained \( \lambda_a = 22.7 \pm 0.3 \) m. For the scattering length, defined as \( \lambda_s = 1/b \), he obtained \( \lambda_s = 23 \pm 1 \) m. Using the result \( \langle \cos \theta \rangle \approx 0.857 \) of the analysis of the angular distribution of scattered light, for the effective scattering length, defined as \( \lambda_e = \lambda_s/(1 - \langle \cos \theta \rangle) \), he obtained \( \lambda_e \approx 160 \) m.

B. Correcting Attenuation Coefficients for Scattering

For the NESTOR and DUMAND sites there exist spectral data on attenuation (discussed in Subsections 3.C and 3.D). At the NESTOR site absorption was measured at depths >3500 m at a single wavelength, \( \sim 450 \) nm. At the DUMAND site absorption was measured only at depths to 1200 m and only at 480 nm. Scattering was not measured at either site. Let us consider how to disentangle absorption and scattering from the attenuation data for these and other deep ocean sites. Mobley\(^{13}\) gives a useful discussion of models for absorption, scattering, and attenuation in natural waters. The beam attenuation coefficient is divided into four parts: \( c(\lambda) = c_w(\lambda) + c_s(\lambda) + c_p(\lambda) + c_{\omega}(\lambda) \), where \( c_w \) is the contribution of optically pure (only H\(_2\)O + dissolved salts) seawater, \( c_s \) is due to chlorophyll and related pigments in phytoplankton, \( c_p \) is due to dissolved organic compounds (typically yellowish), and \( c_{\omega} \) is the attenuation by suspended organic and inorganic particles. At deep ocean sites where sunlight does not penetrate, there is no phytoplankton, and \( c_p \) can be taken to be 0. In addition, measurements\(^{14}\) at various wavelengths have shown that in the deep ocean \( c_p \) is negligible compared with \( c_{\omega} \). The attenuation in excess of that due to pure seawater, \( c - c_{\omega} \), is attributed solely to suspended and settling organic and inorganic particles, partly of aeolian (wind driven) and partly of pelagic origin. Gardner\(^{14}\) has shown that measurements of \( c \) at 660 nm in the deep ocean with a Sea Tech transmissometer are sufficient to characterize particulate scattering reasonably well at all wavelengths if one subtracts \( c_{\omega}(660) \) and uses a power-law wavelength dependence \( \lambda^{-\eta} \), with \( \eta \approx 1–2 \) depending on the size distribution of the suspended particles.

The size distribution \( dN/dr \) of both aerosol particles and suspended particles in the deep ocean can be represented equally well by a log normal, with a modal radius of \( \sim 1 \ \mu \text{m} \)\(^{15,16}\) or by a power law \( dN/dr \propto r^{-s} \), with \( s \approx 4 \) (Ref. 17). I first make an order-of-magnitude estimate of the contribution of the mineral component of such particles to scattering and then develop a more quantitative model for all the particles. The flux of aerosols precipitating onto the ocean surface\(^{18,19}\) is typically 0.2–0.3 mg cm\(^{-2}\) yr\(^{-1}\). The buildup of the aerosol component of the sea sediment is observed to be the same, within errors. Thus most of the insoluble aerosol particles sink through the ocean and reach the bottom. The use of Stokes’ law leads to a settling velocity of 3 \( \mu \text{m} \) s\(^{-1}\) for particles of 1-\( \mu \text{m} \) radius and to a mass concentration of such particles in the deep ocean, \( C_m = 30 \) ng g\(^{-1}\). Of course, this result cannot be accurate in detail because it ignores currents and interactions between particles such as collisions and sticking.

Pelagic (nonaerosol) particles may also contribute to scattering. Experiments with particle traps at various depths have shown that the mass flux of organic and inorganic carbon-bearing particles to the ocean floor is dominated by large sizes and is an order of magnitude greater than that of aerosols.\(^{20}\) However, their settling velocity (typically \( \sim 0.1 \) cm/s)\(^{21}\) is much greater than that of aerosols, and their contribution to light scattering can thus be neglected. The vertical distribution of submicrometer particles caught with Nuclepore filters (radii 0.2–0.5 \( \mu \text{m} \)) has been studied to depths of 3200 m in the northern North Pacific Ocean.\(^{22}\) Their mass concentration decreases to an asymptotic value of \( \sim 6 \) ng/g at depths below a few hundred meters.

The application of Mie scattering theory to spherical particles with a size distribution of \( \sim r^{-4} \) and a rms radius \( \bar{r} \) leads to a scattering coefficient \( b_s \approx \pi \bar{r}^2 Q(r, m) n = 3Q C_m/4 \rho \bar{r} \) [where \( n = \) the concentration of particles, \( \rho = \) the mass density of a particle, \( m = \) the refractive index, and \( Q(r, m) \pi \bar{r}^2 \) = the scat-
tering cross section). For detritus and mineral grains with $m = 1.15$ and $\vec{r} \approx 1 \, \mu m$, I estimate $Q = 2$ and $b_p \approx 0.018 \, m^{-1}$. For biogenic particles with $m = 1.05$ and $\vec{r} \approx 0.35 \, \mu m$, $Q$ is so small that its contribution to scattering can be neglected. By adding the calculated contribution of molecular scattering by seawater, I obtain $b = b_w + b_p \approx 0.02 \, m^{-1}$ at a wavelength of 450 nm. This estimate is uncertain due both to neglect of interactions among settling particles and to the role of bottom currents, which may stir up a suspension of sediments in a time-varying way.

To model more accurately the wavelength dependence of scattering from a distribution of particle sizes, I assume that

$$\hat{b}_p(\lambda) = A(400/\lambda)^{1.7}, \quad \text{(1)}$$

where the exponent 1.7 is taken for small suspended particles in the shallow ocean and applies throughout the wavelength region of good transparency ($400–650 \, nm$). In Subsections 3.C and 3.D, I estimate values of $A$ for the NESTOR and DUMAND sites.

C. NESTOR Site

Some of the earliest measurements of attenuation spectra were made by Matlack, who used a dropped device to make in situ measurements as a function of depth in the Atlantic Ocean and at a few sites in the Mediterranean Sea. More recently Khanaev and Kuleshov collected a large number of water samples from many depths at seven locations at $36^\circ 37.2' \, N$, $21^\circ 29' \, E$, southwest of Greece. They used a highly collimated beam spectrophotometer on shipboard to measure the attenuation coefficient at 16 wavelengths from 310 to 610 nm in water taken at depths ranging from the surface to the bottom (at approximately 4000 m). To a depth of $\approx 1500 \, m$, attenuation decreased, below which it became independent of depth. For various samples, the attenuation reached a minimum at wavelengths of 470–490 nm, at values $c(480) = 0.025$ to 0.045 $m^{-1}$. The open diamond points in Fig. 1 are attenuation coefficients for their clearest water sample [Fig. 2(d) of their paper]. Their results were consistent with the earlier, less complete results of Matlack.

The collection of water samples in plastic bottles has two possible pitfalls: (1) The water may be contaminated during the collection procedure; and (2) a downward flux of particulates is maintained in an approximately steady state while in situ, but changes when the sample is collected and brought on shipboard. Specifically, the heavier particles may sink before the measurement can be made, thus leading to an underestimate of attenuation.

Using an uncollimated photostrobe with a 460-nm interference filter to measure the 1/e transmission distance, Anassontzis et al. obtained values of absorption length at depths greater than 3500 m at three of the sites where Khanaev and Kuleshov took samples. They found an average value of $a = 55 \pm 10 \, m$. I used its reciprocal $a(460) = 0.018 \, m^{-1}$ and subtracted the component $b_w$ that is due to molecular scattering, from which I derived the value $A = 0.016$ in Eq. (1). This enabled me to convert the attenuation spectrum of Khanaev and Kuleshov into the absorption spectrum shown in Fig. 2 as open

![Fig. 2. Values of absorption coefficient obtained by subtracting scattering from measurements of attenuation coefficient (NESTOR24 and DUMAND28) or of diffuse attenuation coefficient (Smith and Baker7).](image)

Fig. 3. Values of absorption coefficient for purest water, South Pole ice at depths of 830 and 970 m, Lake Baikal, and the DUMAND site near Hawaii and the NESTOR site southwest of Greece. The solid curve is my estimate of the absorption spectrum for South Pole ice at a depth of $\approx 2000 \, m$ where the dust concentration is believed to be only approximately 30 ng g$^{-1}$. SNO, Sudbury Neutrino Observatory.
circles and in Fig. 3 as solid square points. It is reassuring (but somewhat accidental, in view of the large errors) that the value \( b_p = 0.015 \text{ m}^{-1} \), estimated in the previous section for settling aerosol + carbon-bearing sediments, is consistent with the value of \( b_p = 0.016 \) estimated from Eq. (1) and subtraction.

D. DUMAND Site

Spectral measurements of attenuation were made at the DUMAND site by Zaneveld and co-workers. Harvey et al.\(^{25}\) made measurements in free fall with a self-contained transmissometer (together with instruments to measure conductivity, temperature, and depth). They reported profiles both at Maui Basin and off Keahole Point (the present DUMAND site). At 650 nm the average value of attenuation length in the deepest 1000 m was 2.24 m at the DUMAND site 45 km west of Keahole Point and 2.20 m at Maui Basin. Zaneveld\(^{28}\) used a cable-mounted transmissometer with a path length of 1 m, a pulsed xenon light source, and filters to profile the beam attenuation at 50-nm intervals from 400 to 650 nm at depths to 4567 m at the DUMAND site. He found a decrease in attenuation at all wavelengths as a function of depth, leveling off in the bottom \( \sim 1000 \) m. His data, averaged over the lowest 1000 m, are shown as solid triangles in Fig. 1. Figure 2 shows his data (solid triangles) corrected for scattering as described below and in Subsection 3.B.

In addition, Zaneveld took time-series data at 660 nm with transmissometers moored at a depth of 10 m above the ocean floor at two positions separated by 5 km, one of them in the Maui Basin and one west of Keahole Point. Results for the various experiments were consistent with each other at comparable wavelengths. As a result of settling of particulate matter onto the optics, both transmissometers showed a steady decrease in transparency with time. To help interpret the time dependence, Zaneveld collected water samples and measured the size distribution and mass concentration of particulates in the radius interval between \( -0.8 \) and \( -16 \) \( \mu \text{m} \). By integrating over a radius distribution \( r^{-3} \), between the limits 0.02 and 20 \( \mu \text{m} \), and taking into account the size dependence of the Stokes' settling velocity, he estimated a rate of fractional area coverage of \( 2.8 \times 10^{-3} \) per day, which was not inconsistent with the data. His mass concentration ranged from 20 to 60 ng/g. Assuming the same correlation between mass concentration and beam attenuation found for diatomic Earth, he inferred the contribution of particles to attenuation to be 0.007–0.02 m\(^{-1}\), not including the contribution of particles with a radius \(<0.8 \mu\text{m}\).

Andrews et al.\(^{29}\) reported a measurement of \( \sim 30 \) m for the attenuation length near the sea floor in the blue-green (\( \sim 460 \) m). This is consistent with the value \( 0.037 \text{ m}^{-1} \) obtained by Zaneveld\(^{28}\) at 450 nm. They also reported a near-bottom nepheloid layer at Keahole Point, a decrease of transmission with time that was due to sedimentation and a particulate concentration of 20–50 ng/g near the sea floor.

Robertson\(^{30}\) refers to a measurement of \( 47 \pm 22 \) m at a wavelength of 410 nm for the attenuation length in the deep ocean 30 km west of Keahole Point, but none of his references describes this work.

In his Ph.D. dissertation Clem\(^{31}\) devised two methods for estimating the effective attenuation coefficient at a single wavelength of \( \sim 415 \) nm and reported a value of \( c = 0.024 \pm 0.009 \text{ m}^{-1} \). Because his results have often been cited by members of the DUMAND collaboration in preference to the earlier measurements of attenuation at a number of wavelengths by Zaneveld, one should take note of the following deficiencies in his results. In his first method, because of the poor geometry of his equipment (a large-angle emitter and receivers), the contribution of scattering to the attenuation was quite uncertain. Based on Monte Carlo modeling, he inferred a scattering length of \( 35–75 \) m and an effective absorption length of \( \sim 40 \) m, but with unknown errors. The geometry of his equipment permitted roughly as much light to be scattered into the path leading to the detector as out of the path leading to the detector. His second method was to calculate the expected intensity of light reaching a phototube from radioactive decay of \( ^{40}\text{K} \) in the ocean integrated over all distances. He incorrectly assumed that the integrated intensity should be proportional to \( \exp(-cr) \), whereas it can be shown that the intensity is proportional to \( \exp(-ar) \) and thus that the absorption coefficient, not the attenuation coefficient, is \( \sim 0.024 \text{ m}^{-1} \). This change makes his result consistent with that of Zaneveld.\(^{28}\)

The only report of an absorption measurement was that of Bradner and Blackinton,\(^{25}\) who used an uncollimated photostrobe with an interference filter to measure the \( 1/e \) transmission distance of 480-nm light at the DUMAND site 34 km west of Keahole Point. From measurements at two source distances, 7.86 and 84 m, they obtained a value of \( 25 \pm 1 \text{ m} \) at a depth of 1200 m, which they interpreted as the absorption length.

Lacking absorption data near the ocean floor at the DUMAND site, I used the NESTOR value \( A = 0.016 \text{ m}^{-1} \) in Eq. (1) to obtain the absorption spectrum shown in Figs. 2 and 3 from the DUMAND attenuation spectrum in Fig. 1. From the smooth curve through the data points in Fig. 2, one can see that, within errors, the absorption spectra for DUMAND and NESTOR are indistinguishable, and the absorption for these two sites is greater than that for Smith and Baker’s ideal seawater at wavelengths shorter than \( \sim 450 \) nm and consistent with Smith and Baker at longer wavelengths.

E. General Results for Deep Pacific Ocean Sites

Using a Sea Tech transmissometer together with an instrument to measure salinity, temperature, and pressure, Gardner et al.\(^{32,33}\) and Colgan\(^{34}\) have determined \( b_p \) at 660 nm from the surface to the ocean floor as a function of latitude and longitude over large regions of the Pacific Ocean. The value of \( b_p \) is inferred by subtraction: \( b_p(660) \approx c_p(660) = c(660) – c_{\text{salt}}(660) \).
c_in(660). From their data, one can draw several conclusions relevant to the optical clarity of the deep Pacific Ocean:

- Due to near-bottom nepheloid layers, the clearest region is almost always somewhat above the ocean floor.
- Despite daily and annual variations in particle concentration, there exist many potential sites for an array of phototubes in an ~1-km region nearest the ocean floor where the contribution to scattering by particles is as low as 0.005–0.01 m\(^{-1}\) and wavelength dependence from \(\lambda^{-1}\) to \(\lambda^{-2}\).
- At these favorable sites, although scattering by particulates dominates over molecular scattering at all wavelengths of interest, scattering by particulates makes only a minor contribution to the degradation of information about charged particle trajectories when account is taken of the angular distribution of scattered light.

What is still lacking in oceanographic data are direct measurements of the wavelength distribution and the angular distribution of scattering in deep ocean water. Figure 4 shows my attempt to take both effects into account so as to be able to estimate the contributions of molecular and particulate scattering as a function of wavelength. In the figure, I converted the scattering coefficient to an effective scattering coefficient by multiplying by an angular factor

\[
b_{\text{e}} = \Sigma b_{\lambda}(\lambda)(1 - \tau),
\]

where \(\tau = \langle \cos \theta \rangle\) averaged over all the distribution of scattering angles and the subscript designates the type of scattering (molecular or particulate). The relative importance of molecular scattering is increased as a consequence of the forward–backward symmetry, which gives \(1 - \tau = 1\), in contrast to the very small value of \(1 - \tau_p \approx 0.1\) or less. The dashed curves bracket the expected values of \(b_{\lambda}(\lambda)\) for a wavelength dependence ranging from \(\lambda^{-1}\) to \(\lambda^{-2}\).

4. Optical Properties of Deep Ice

The AMANDA Collaboration used a pulsed laser technique to determine separately the absorption and scattering of light at ten wavelengths from 410 to 610 nm at several depths from 830 to 1000 m in ice at the South Pole.\(^3\) Figure 3 shows the dependence of absorption coefficient \(a(\lambda)\) on wavelength for ice at depths of 830 and 970 m. The change in \(a(\lambda)\) with depth at the shortest wavelengths is significant and is interpreted\(^{35,36}\) as due to a contribution to absorption in ice by insoluble dust that increases in concentration with depth in the interval 830–970 m. (A dust band corresponding to the last ice age, at \(\sim 17,000\) years before the present era, is predicted to begin at \(\sim 1000\) m with a thickness of \(\sim 100\) m. The location of the dust band accounts for the increase in the absorption.)

The solid curve in Fig. 3, discussed in Ref. 36, is the rough prediction by the AMANDA Collaboration of the absorption by deep ice with a dust concentration of approximately 15 ng g\(^{-1}\), corresponding to depths \(>2000\) m. When absorption in the glass pressure vessel and phototube quantum efficiency are taken into account, the wavelength interval of concern to us is 320–620 nm. At wavelengths for which the absorption is near its minimum value, the pulsed laser technique\(^3\) gives values an order of magnitude lower than those measured\(^{37}\) for laboratory ice. The discrepancy is almost certainly due to failure in the latter case to take into account scattering; i.e., the latter data pertain to attenuation rather than to absorption. Measurements now being made by the AMANDA Collaboration in ice at depths to 2.0 km (Ref. 4) will provide information on both \(a(\lambda)\) and \(b_{\lambda}(\lambda)\) at various dust concentrations and at wavelengths from 610 to 337 nm.

Measurements\(^3\) showed that, at depths of 830–1000 m, scattering is due predominantly to frozen-in air bubbles, with values of \(b_{\lambda}(\lambda)\) decreasing from \(\sim 10\) to \(\sim 5\) m\(^{-1}\) with depth, because of the increase in the fraction of bubbles converted to air-hydrate clathrate crystals with increasing depth. A model\(^{38}\) of the kinetics of the phase transformation can be used to predict that, at depths below 1500 m, all the bubbles will have transformed into hydrate crystals. Scattering at such depths is expected to be due mainly to dust and liquid acids, with minor contributions that are due to ice–ice crystal boundaries and to ice–hydrate crystal boundaries.

5. Comparison of Optical Parameters of Water and Ice at the Various Sites

Table 1 displays values of the wavelength at which absorption is a minimum, the minimum value of the absorption coefficient, and the various contributions to scattering at 450 nm. The expressions for propagation of light always include the quantity \(b_{\lambda}\), which takes into account the angular distribution for a particular mode of scattering. For smooth bubbles with
a diameter much greater than a wavelength, \( \tau_b = 0.75 \); for molecular scattering, \( \tau_m = 0 \); and for scattering from particulates with size of the order of the wavelength immersed in ice or water, the scattering is strongly forward peaked and \( \tau_p \) is from \(-0.9\) to \(0.96\). The last column gives estimates for the propagation coefficient

\[
\alpha = \{3\alpha(\lambda)\Sigma(b_i(\lambda)(1 - \tau_i))\}^{1/2}. \tag{3}
\]

This coefficient appears in Eq. (4), the expression for the number of photons per unit area reaching a phototube at a distance \(d\) after diffusive propagation from a point source through a medium that both scatters and absorbs\(^{36}\):

\[
I(d) = \frac{3N_b}{16\pi d} \exp(-\alpha d). \tag{4}
\]

Here \(N\) is the number of photons injected at a point. It is interesting to compare the magnitudes of the product \(b_i(\lambda)(1 - \tau_i)\) and of \(\alpha\) for various entries in Table 1. For shallow ice, scattering takes place dominantly from bubbles; for deep bubble-free ice, light is scattered mainly from impurities such as dust and veins of liquid acids, even at wavelengths as short as 330 nm; for deep ocean sites such as DUMAND and NESTOR, molecular scattering contributes more than does scattering from particulates; and for Lake Baikal, both molecular and particulate scattering make comparable contributions. As in Fig. 4, for natural waters the highly forward-peaked scattering from particulates weakens their role relative to molecular scattering. The parameter \(\alpha\) (column 8) is least favorable (largest) for bubbly ice, most favorable for ice at great depths where bubbles are absent and the dust concentration is lowest, and intermediate for water. The DUMAND and NESTOR sites are better than either Lake Baikal or high-dust, bubble-free ice.

### 6. Effective Volume per Module as a Function of Cascade Energy for Interactions of Electron Neutrinos

A high-energy neutrino-detecting array uses phototubes to record the weak visible Cherenkov radiation emitted in a clear medium traversed by the charged particle into which the neutrino has converted. The direction of emission is along a cone with half-angle \(\arccos(c/mv)\), where \(m\) is the refractive index. We now focus attention on the cone of Cherenkov light emitted by the large number of charged particles in the electromagnetic or nuclear cascade produced when an electron–neutrino \(\nu_e\) interacts with an electron or a nucleus in water or ice. The number of Cherenkov photons emitted per unit wavelength for electrons plus positrons in the cascade is given by the product \(L(E_\nu) \times dN/dLd\lambda\), where \(L(E_\nu)\) is the total path length of electrons plus positrons and \(dN/dLd\lambda\) is the number of photons per unit length per unit wavelength,

\[
dN/dLd\lambda = 2\pi\alpha_e(1 - \beta c^2 m^2)\lambda^{-2}. \tag{5}
\]

Here \(\alpha_e\) is the fine-structure constant, \(\beta c = c\) is the speed of electrons and positrons, and \(L(E_\nu)\) is found experimentally to be proportional to the total energy of the cascade: \(L(E_\nu) \approx (6400 \text{ m/TeV})E_\nu(\text{TeV})\).\(^{39}\)

Because the typical distance \(d\) from cascade to phototube is much greater than the length of the cascade (36 g cm\(^{-2}\) for an electromagnetic cascade, 85 g cm\(^{-2}\) for a hadronic cascade), one can regard the cascade as occurring at a point. (The unit of length in g cm\(^{-2}\) is convenient when the medium may have a varying density but the same composition, such as ice and water.) Reference 40 gives a thorough discussion of the process and presents rough estimates of \(V_{\text{eff}}\), the effective volume per module, with simplifying assumptions: a constant value of \(\alpha(\lambda)\) over the band 350–600 nm, a constant quantum efficiency \(\eta = 25\%\) over this band, and a constant value \(T(\lambda) = 1\) for the fraction of light transmitted through the pressure vessel. To determine \(V_{\text{eff}}\) as a function of energy more accurately than in Ref. 40, I take into account the wavelength dependence of \(T(\lambda)\), \(\eta(\lambda)\), \(\alpha(\lambda)\), \(b(\lambda)\), and \(\tau(\lambda)\) for the various sites proposed for high-energy neutrino observatories. As the first step in determining \(V_{\text{eff}}\), I calculate, as a function of distance, the number of photons \(N_{\text{PMT}}\) recorded in a photomultiplier tube of area \(A_{\text{PMT}}\) for two limiting cases. Equations (6) and (7) are extensions of expressions derived in Ref. 40. Equation (6) is applicable if the

### Table 1. Comparison of Optical Properties of Various Sites

<table>
<thead>
<tr>
<th>Site</th>
<th>(\lambda_{\text{min}}) (nm)</th>
<th>(\alpha_{\text{min}}) (m(^{-1}))</th>
<th>(b_0(1 - \tau_p))</th>
<th>(b_0(1 - \tau_m))</th>
<th>(b_0(1 - \tau_b))</th>
<th>(\Sigma N_i(1 - \tau_i))</th>
<th>(\alpha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bubbly ice at 830 m</td>
<td>-360(^b)</td>
<td>0.002(^b)</td>
<td>7 \times 10^{-4} \times 1</td>
<td>0.04 \times 1</td>
<td>2.5</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>South Pole ice in low-dust region</td>
<td>-360(^b)</td>
<td>0.002(^b)</td>
<td>7 \times 10^{-4} \times 1</td>
<td>0.04 \times 1</td>
<td>2.5</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>South Pole ice at 1.7 km(^c)</td>
<td>-360</td>
<td>-0.01</td>
<td>7 \times 10^{-4} \times 1</td>
<td>-0.2 \times 0.2</td>
<td>0.04</td>
<td>0.035</td>
<td></td>
</tr>
<tr>
<td>Pure seawater</td>
<td>430</td>
<td>0.0144</td>
<td>0.0045 \times 1</td>
<td>-0</td>
<td>0.15 \times 0.05</td>
<td>0.053</td>
<td>0.017</td>
</tr>
<tr>
<td>DUMAND and NESTOR</td>
<td>480</td>
<td>0.018</td>
<td>0.0045 \times 1</td>
<td>0</td>
<td>0.0155 \times 0.05</td>
<td>0.053</td>
<td>0.017</td>
</tr>
<tr>
<td>Lake Baikal(^{13})</td>
<td>500</td>
<td>0.044</td>
<td>0.0034 \times 1</td>
<td>0</td>
<td>0.043 \times 0.143</td>
<td>0.0096</td>
<td>0.036</td>
</tr>
</tbody>
</table>

\(^a\)Subscripts are for molecular, bubble, particulate, and effective scattering.

\(^b\)Estimated rather than based directly on measurement.

\(^c\)Ref. 4.
distance is so short that almost no scattering occurs ($d \ll 1/b$):

$$N_\gamma^{\text{PMT}} = L(E_\gamma) \frac{1}{\pi} \int T(\lambda) \eta(\lambda) 2 \pi \alpha(1 - 1/\beta^2) \, d\lambda,$$

whereas Eq. (7) is applicable if $r \gg 1/b$ (threedimensional random walk through scattering centers with absorption):

$$N_\gamma^{\text{PMT}} = L(E_\gamma) \frac{1}{\pi} \int T(\lambda) \eta(\lambda) 6 \pi \alpha(1 - \beta^2) \, d\lambda,$$

$$\times \frac{A_{\text{PMT}}}{4 \pi d^3} \exp[-\alpha(\lambda)d] \, d\lambda.$$

In Eqs. (6) and (7), the geometry factor $1/\pi$ takes into account the average orientation of a randomly oriented phototube with respect to the shower axis. The integrals in Eqs. (6) and (7) are taken over wavelengths from 350 to 650 nm.

I define $V_{\text{eff}}$ as the spherical volume $4\pi d_{\text{max}}^3/3$, corresponding to the distance $d_{\text{max}}$ at which the number of detected photons $N_\gamma^{\text{PMT}}$ is only 1. The simplest way to determine $V_{\text{eff}}$ as a function of $E_\gamma$ from Eqs. (6) and (7) is to replace $L(E_\gamma)$ by $6400E_\gamma$ in Eqs. (6) and (7) and to solve for $E_\gamma$ for various values of $d = d_{\text{max}}$. Figure 5 shows results obtained by smoothly joining Eqs. (6) and (7), with values of $a(\lambda)$, $b_i(\lambda)$, $\tau_i$, and $\alpha(\lambda)$ taken from Table 1. All the curves in Fig. 5 are calculated for an 8-in. (20-cm)-diameter phototube with $A_{\text{PMT}} = 0.028 \text{ m}^2$. Scaling to other diameters is trivial.

7. Effective Volume for Detection of Few-Megaelectron Volt Neutrinos from a Supernova

Halzen et al. have shown that the AMANDA array at the South Pole is an effective system for recording the burst of neutrinos from a nearby supernova (SN). The signal in a phototube comes from the Cherenkov photons emitted by an $\sim 20$-MeV positron released when an electron–antineutrino is captured by a proton in the ice. We seek an expression for $V_{\text{SN}}$, the effective volume of ice within which the positron would be detected. The passage through the Earth of a huge flux of several megaelectron volt neutrinos during a period of less than approximately 10 s can be detected as an excess of single counting rates in all individual phototubes. Here one is simply searching with a large number of phototubes for statistically significant evidence for the burst of neutrinos. It is interesting that, even in bubbly ice, the method is expected to work.

To obtain $V_{\text{SN}}$ involves integrating Eq. (7) over all volume. I note that this procedure greatly overestimates the effective volume for signals such as muon tracks or electromagnetic cascades whose trajectories must be studied by recording counts in each of a number of phototubes.

As long as the diffusion distance traversed by a photon in reaching a phototube is shorter than the absorption distance, only the absorption coefficient enters into the calculation of effective volume. The result of an integration over all space is

$$V_{\text{SN}} = \frac{6400(E_\gamma) A_{\text{PMT}}}{4} \frac{2\pi \alpha(1 - \beta^2)}{\lambda^2} \int \frac{\eta(\lambda) T(\lambda)}{\alpha(\lambda)} \, d\lambda. \quad (8)$$

(The exponential absorption drops out when one integrates over all volume, leaving $V_{\text{SN}}$ proportional to $1/a$.) With mean positron energy $\langle E_\gamma \rangle = 20$ MeV,
the track length in Cherenkov photons is 0.13 m. Figure 6 displays the results of the integrations as a function of maximum absorption length (or minimum absorption coefficient), taken from the spectra in Fig. 3.

8. Discussion

The entries in Table 1, together with Figs. 5 and 6, enable one to compare the suitability of the various sites for ultrahigh-energy neutrino astrophysics, as far as optical properties are concerned:

- According to Eq. (8), the effective volume $V_{SN}$ for detection of a burst of neutrinos from a supernova or a gamma ray burster is proportional to $a^{-1}$. From column 3 of Table 1 and from Fig. 6 one can see that, with its low absorption, ice is much superior to ocean and lake sites.

- According to Eq. (7), a medium with a low value of $\alpha$ and a high value of $b_\alpha$ is favored for detection of $\nu_\mu$-induced cascades. Bubble-free, dust-free ice is calculated to have the smallest $\alpha$ and the largest $V_{eff}$. Ice with sufficiently low dust concentration is expected to exist at depths below $\sim 2$ km. Deep ocean water in favorable locations may be next in terms of $V_{eff}$ followed by deep South Pole ice with intermediate dust concentrations and by Lake Baikal.

- For detection of $\nu_\mu$-induced muons, not only the transparency but also the ability to determine muon trajectory are important because one must discriminate against downward-going atmospheric muons, and one would like to see point sources of $\nu_\mu$. The magnitude of the quantity $b_\mu = \sum b_\nu(1 - \tau_\nu)$ (Table 1, column 7) limits the accuracy of muon trajectory determination. The DUMAND and NESTOR sites and bubble-free, dust-free ice would seem to be best. Because of its small intrinsic (molecular) scattering, ice would be the clear winner if it were not for the dust and possibly liuid acids. Ongoing measurements of $b_\mu(1 - \tau_\mu)$ as a function of depth in South Pole ice are crucial. For example, if $b_\mu$, at $\sim 2000$-m depth turns out to be $\geq 0.1$ m$^{-1}$ instead of the assumed value 0.04 m$^{-1}$, deep ice would be significantly worse than the clearest ocean sites. For the best water sites, molecular scattering seems to be the limiting factor. However, if bottom currents give rise to a time-dependent nepheloid layer, or if particulates other than those of aeolian origin are sometimes important, ocean water could be significantly worse than ice.

As a consequence of our doing an integration over wavelength rather than making simplifying assumptions, and of finding values of attenuation and absorption reported in the (unrefered) literature, the results shown in Fig. 3 for effective volume as a function of cascade energy differ in some respects from those in Ref. 40. Taking wavelength dependence of quantum efficiency of the phototube into account favors ice over water, because for ice the minimum in the absorption curve corresponds closely to the maximum in quantum efficiency. The effective volumes in Fig. 3 are larger for ice and smaller for water than the ones in Ref. 40.

For ocean sites the necessity to infer the absorption spectrum from an attenuation spectrum by subtraction introduces large errors. An additional uncertainty results from the limited knowledge of the size distribution and nature of the particulate scatterers in both ice and water. Although extensive data on grain size distribution for mineral grains in abyssal sediments exist, information on distributions of non-mineral particles in the deep ocean is more sparse.

To carry out calculations of effective volume for $\nu_e$-induced cascades and for muon signals, and thus to characterize the performance of an ultrahigh-energy neutrino detector array, one needs to know separately the absorption and scattering, as well as the angular distribution of scattered light. At the South Pole site the elegant pulsed laser technique provides both absorption and scattering as a function of wavelength and depth. Measurements planned for wavelengths $\sim 300-400$ nm are essential. Data on both absorption and scattering (the latter with large errors) have been obtained for Lake Baikal. At the DUMAND and NESTOR sites, despite the availability of extensive data on attenuation, no data on scattering have yet been obtained, and there is little information on absorption. Direct measurements with the technique used by the AMANDA Collaboration should be considered. Crawford et al. have discussed a design for an autonomous instrument that could be dropped from a ship and would detect single photons with precise timing, from which both scattering and absorption could be measured.

The use of an integration over wavelength to calculate the effective volume for supernova detection in bubbly ice leads to a value about half as large as that calculated with simplifying assumptions. Because upward-looking instruments at both the DUMAND site and the Lake Baikal site suffer a decrease of effective surface area with time as a result of sedimentation, and because sedimentation occurs in all natural bodies of water, it seems clear that upward-looking phototubes in lakes or oceans will degrade with time. The tentative design of NESTOR has half of the phototubes pointing upward. Some scheme should be devised for removing the sediments that will inevitably build up. The problem of contamination of phototube surfaces does not arise in the case of ice.

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