EHE hadronic shower parametrization

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(Dated: October 16, 2006)

The hadronic shower parametrization (longitudinal and radial) used for the SAUND project [1, 2], and for related acoustic neutrino signal simulations [3], is presented.

The hadronic shower parametrization we use is the NKG parametrization presented in [4], with the following features:

1 We normalize to the total shower energy.

2 We use energy-dependent tail length and maximum shower depth parametrized from simulations that include the LPM effect [5].

As presented in [4], the particle density in a hadronic shower as a function of depth X and radius r can be approximated by the Nishimura-Kamata-Greisen parametrization,

$$D(X, r) = N(X)\rho(r), \tag{1}$$

where

$$N(X) = N_{max}((X - X_0)/(X_{max} - X_0))^{[(X_{max}-X_0)/\lambda]} \exp\left[(X_{max} - X)/\lambda\right], \tag{2}$$

$$\rho(r) = (N_0/(r_1)^2)f(s, r/r_1), \tag{3}$$

and

$$f(s, r/r_1) = (r/r_1)^{s-2}(1 + r/r_1)^{s-4.5}\Gamma(4.5 - s)/[2\pi\Gamma(s)\Gamma(4.5 - 2s)]. \tag{4}$$

Here N_{max} is the maximum particle density, X_{max} is the depth at which it occurs, $\lambda = 70$ g/cm^2 is the tail length, r_1 is the Moliere radius, and s is the effective age, 1.25. For the Moliere radius, we use $r_1 = 0.021 X_{rad}/E_{crit}$, where the radiation length $X_{rad} = 0.3608$ m and the critical energy $E_{crit} = 0.0838$ GeV. Choosing a coordinate system where X_0 (the interaction depth) = 0, and defining $t = X_{max}/\lambda$,

$$N(X) = N_{max}(X/X_{max})^t \exp[t - X/\lambda] \tag{5}$$

We wish to renormalize such that the volume integral over the distribution is not the total number of particles but is unity:

$$\int D(X, r) 2\pi r dr dX = 1. \tag{6}$$

Then we can multiply by either total number of particles (or total shower energy) to get the particle density (or energy density). The radial part is already normalized,

$$\int \rho(r) 2\pi r dr = 1, \tag{7}$$

if we choose

$$N_0 = 1. \tag{8}$$

So it remains to normalize

$$\int N(X) dX = 1. \tag{9}$$

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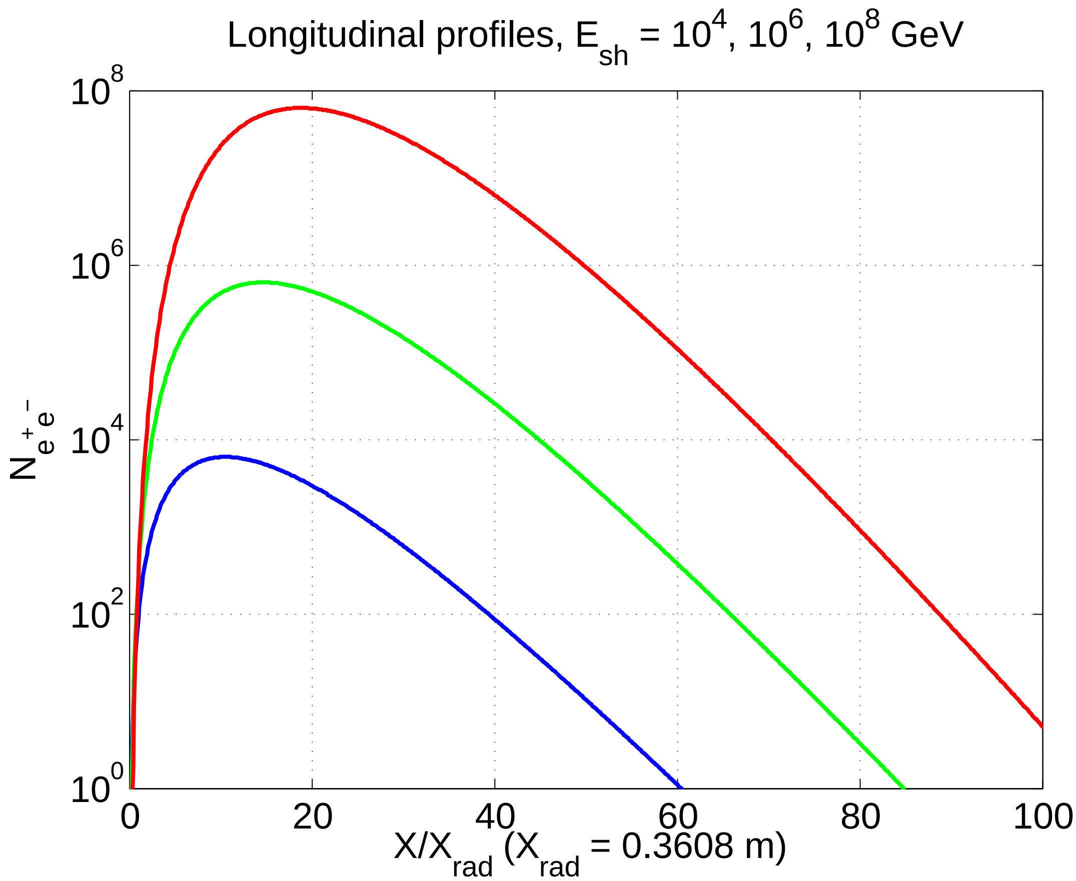

FIG. 1: Longitudinal distributions.

This is achieved by choosing

$$N_{max} = t^{t-1}/[\exp(t)\lambda\Gamma(t)]. \tag{10}$$

Results from simulating hadronic showers including the LPM effect are presented in [5] for $E_{sh} = 10^4, 10^6$, and 10^8 GeV. We parametrize their results as follows:

$$X_{max} = 0.9 X_{rad} \ln[E_{sh}/E_{crit}]; \tag{11}$$

$$\lambda = (1/100)(130 - 5\log[E_{sh}/(10^4\ GeV)])\ m. \tag{12}$$

We can compare this parametrization directly to Fig. 2 of [5]. First we normalize to total number of particles (rather than to unity as given by Eq. (10)) by observing from Fig. 2 of [5] that

$$\log[N_{max}] \approx \log[E_{sh}/GeV] - 0.2. \tag{13}$$

See Fig. 1 for our parametrization with this normalization, to be compared to Fig. 2 of [5]. Fig. 2 gives the longitudinal distribution for a wider range of energies. Figures 3, 4, and 5 give the radial distributions at $10^4, 10^7$, and 10^{10} GeV. Integrating the radial distribution at a particular depth gives dE/dX at that depth.

[1] N. G. Lehtinen, S. Adam, G. Gratta, T. K. Berger, and M. J. Buckingham, Astropart. Phys. **17**, 279 (2002), astro-ph/0104033.

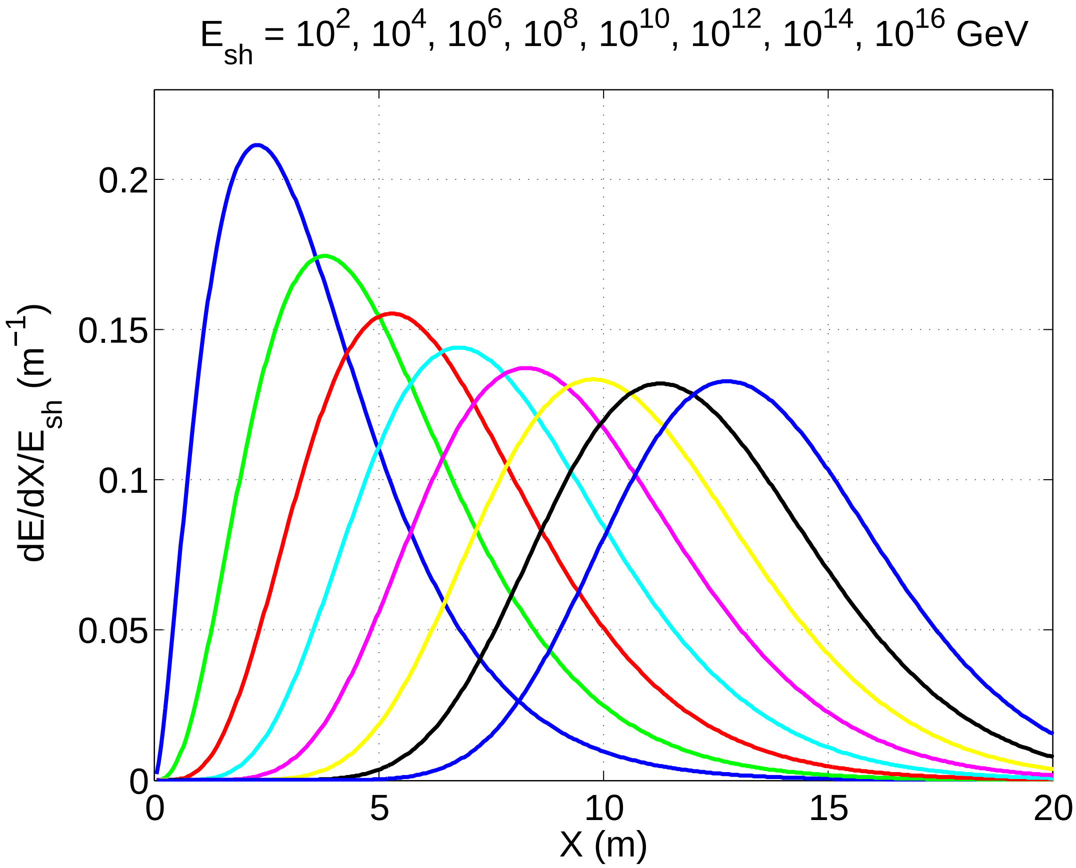

FIG. 2: Longitudinal distributions. Note $\int [dE/dX]dX = E_{sh}$.

[2] J. Vandenbroucke, G. Gratta, and N. Lehtinen, Astrophys. J. **621**, 301 (2005), astro-ph/0406105.

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[4] P. Sokolsky, *Introduction to Ultrahigh Energy Cosmic Ray Physics* (Westview Press, Boulder, CO, 2004), ISBN 0-8133-4212-0.

[5] J. Alvarez-Muniz and E. Zas, Phys. Lett. **B434**, 396 (1998), astro-ph/9806098.

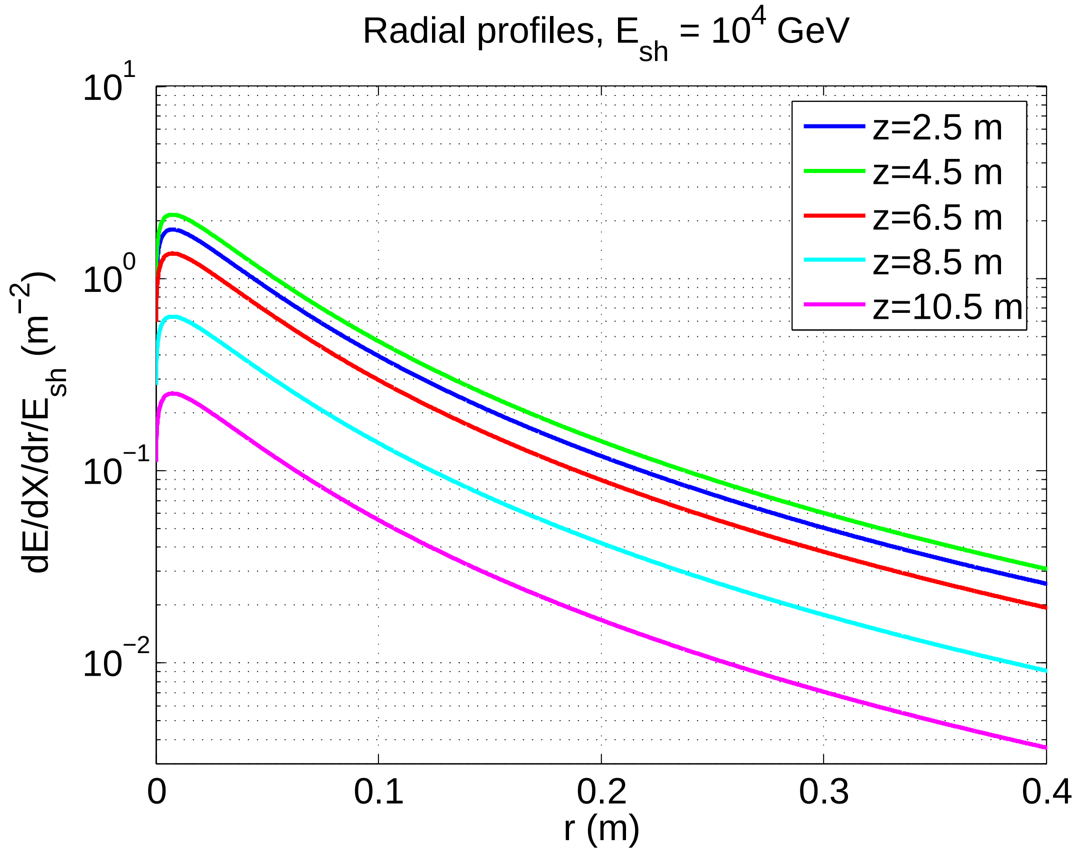

FIG. 3: Radial distribution for 10^4 GeV. Note $\int [dE/dX/dr]dXdr = E_{sh}$.

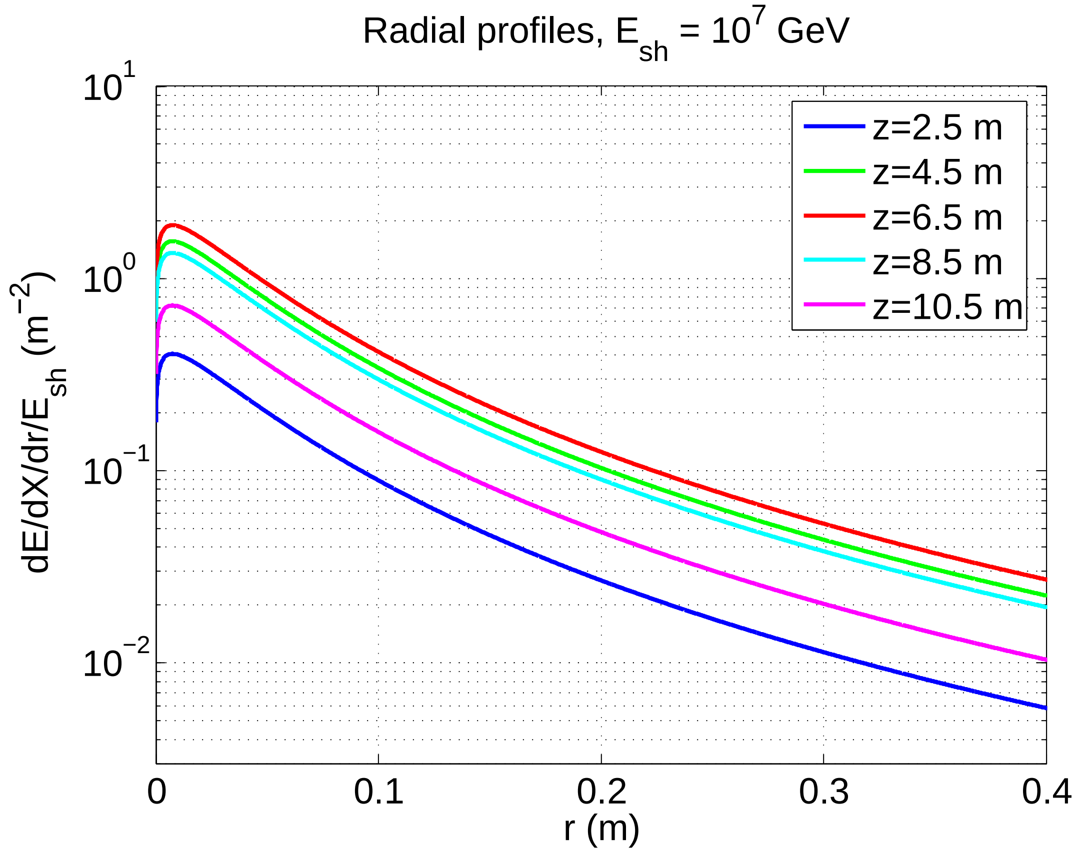

FIG. 4: Radial distribution for 10^7 GeV. Note $\int [dE/dX/dr]dXdr = E_{sh}$.

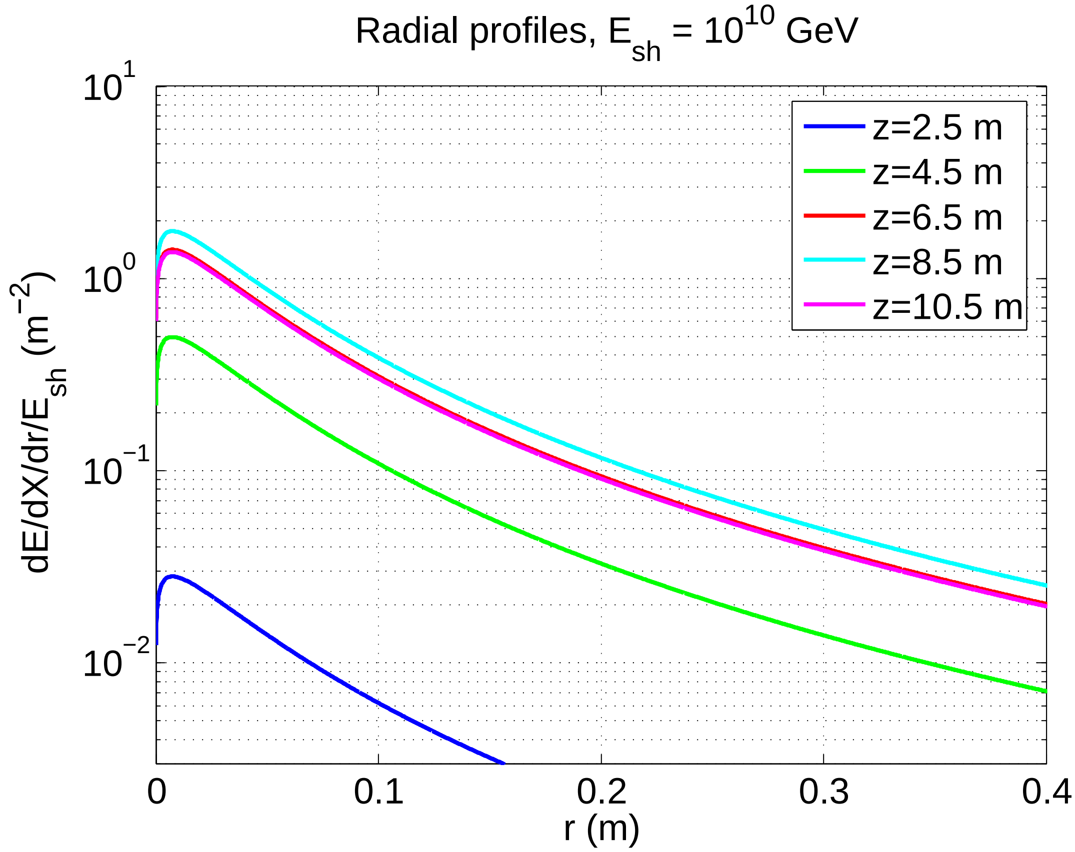

FIG. 5: Radial distribution for 10^{10} GeV. Note $\int [dE/dX/dr] dX dr = E_{sh}$.